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Some Remarks on the Relation $P\sqrt{\varrho}$ for Spectroscopic Binaries and Eclipsing Binaries.

By

E. A. Kreiken.

With 3 figures.

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The period of spectroscopic binaries and of eclipsing binaries is determined by the relation $\log P = -0.5 \log \varrho_t - 0.225$ where ϱ_t is the density of the tenuous component. This relation can be explained from Kepler's third law. For the systems V Crt and TU Mus the dimensions, as derived from the photometric elements, lead to controversial results.

With the early type stars the periods of axial rotation are of the same order of magnitude as the periods of orbital revolution of the binaries of the same density.

The Relation Between Period and Density.

When considering [4] the spectroscopic binaries with known dimensions from the "Fifth Catalogue of Spectroscopic binaries" by J. H. Moore and F. J. Neugebauer [6] it was found that the periods of these binaries could be represented by a relation of the form

$$\log P = A \log \varrho_t + B \quad (1)$$

where ϱ_t is the density of the tenuous component. From the 31 pairs which were available, the coefficients A and B were obtained by a least squares solution. The result was

$$\log P = -0.542 \log \varrho_t - 0.301 \quad r = \pm 0.147 \quad n = 31 \quad (2)$$

where r is the probable error of a single deviation $\Delta = \log P_0 - \log P_c$. Recently an extensive catalogue of eclipsing variables has been published by S. Gaposchkin [3], while data concerning eclipsing variables, brighter than photographic magnitude 8.5 at maximum, have been collected by L. Plaut [7].

For 273 of his systems Gaposchkin gives the masses and diameters of both components and therefore the densities are also known. In Fig. 1 these densities are plotted against the corresponding periods. If for these 273 pairs the coefficients A and B are determined by a least squares solution, we find:

$$\log P = -0.464 \log \varrho_t - 0.138 \quad r = \pm 0.201 \quad n = 273 \quad (3)$$

The curve 3 also is indicated in figure 1. The Bravais coefficient of correlation is found to be —0.87.

From the stars, listed by Plaut, 82 pairs can be used for our present purpose. From these we obtain

$$\log P = -0.464 \log \varrho_t - 0.155 \quad r = \pm 0.245 \quad n = 82\,. \tag{4}$$

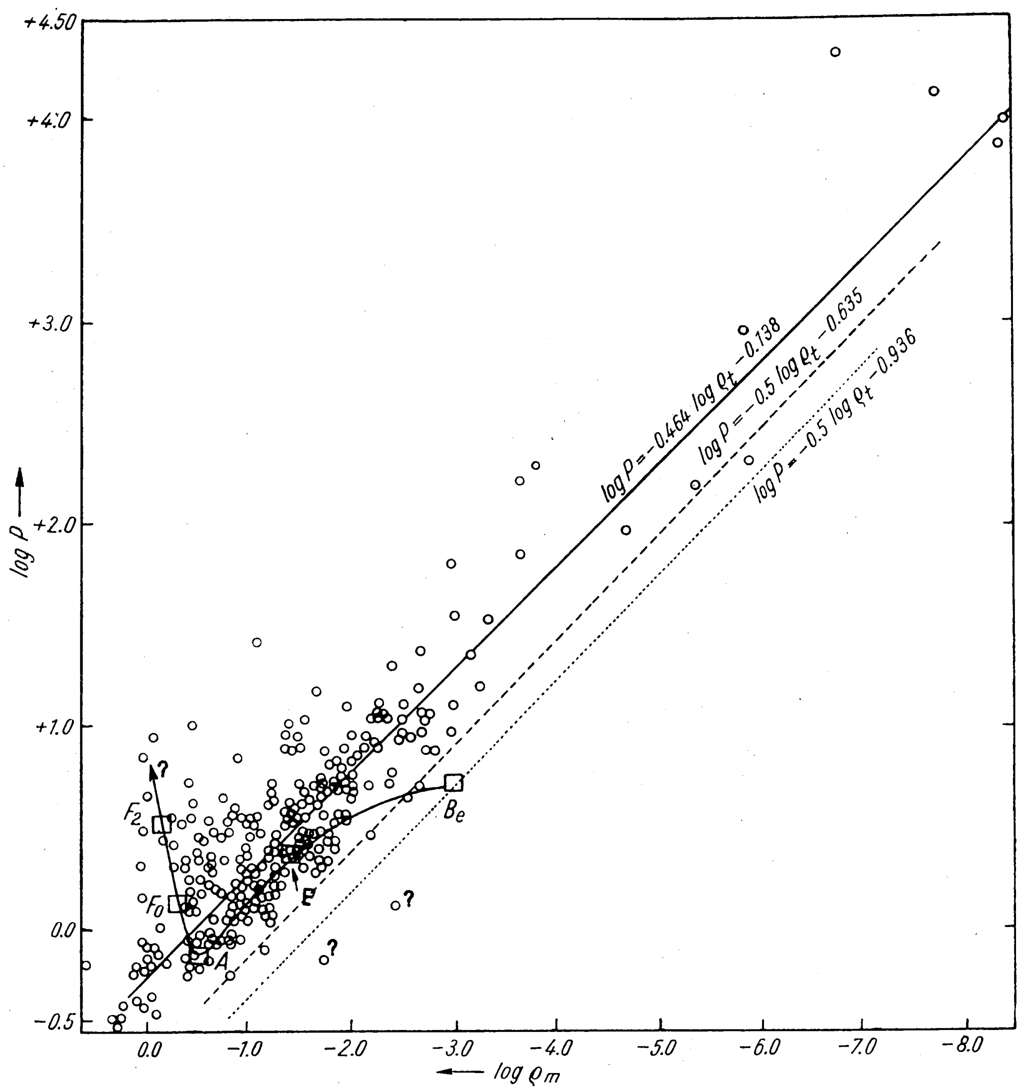

Fig. 1. For all binaries in Gaposchkin's list the logarithms of the period are plotted against the logarithms of the mean density of the tenuous component. The mean relation $\log P = -0.464 \times \times \log P - 0.138$ is indicated by the full drawn line. The broken line indicates the log of period of a pair with two equal components both of density ϱ. Dotted line indicates period of revolution of point m ass on surface of star. Of two stars, V Crt and TU Mus, the position in the diagram is abnormal.

The lists of Gaposchkin and Plaut have many pairs in common and also contain most of the stars enumerated by Moore and Neugebauer. The solutions 2, 3 and 4 therefore are not independent from one another.

Comparison with KEPLER's *Third Law.*

In a previous paper [4] I tried to find some connection between the relation (1) and the Roche limit of axial rotation, but the result was rather negative. The relation can however be explained from KEPLER's third law which we write in the form

$$\mathfrak{M}_t + \mathfrak{M}_d = a^3/74.3\ P^2 . \tag{5}$$

Writing $a = r_t + r_d + s$ it is easy to see that (5) can be written in the form

$$\log P = -\frac{1}{2}\log \varrho_t + \frac{3}{2}\log\left(1 + \frac{r_d}{r_t} + \frac{s}{r_t}\right) - \frac{1}{2}\log\left(1 + \frac{\mathfrak{M}_d}{\mathfrak{M}_t}\right) - 0.936. \tag{6}$$

The subscripts t and d refer to the tenuous and the dense component respectively, while P is in days. If

$$r_d = r_t; \quad \mathfrak{M}_d = \mathfrak{M}_t \quad \text{and} \quad s = 0$$

the equation (6) reduces to

$$\log P = -\tfrac{1}{2}\log \varrho_t - 0.635 \tag{7}$$

and this obviously represents the period of revolution around one another of two identical stars when in contact. In fig. 1 the curve 7 is indicated by a broken line.

If we let both r_d and $\mathfrak{M}_d$ shrink to a very small value, keeping $s = 0$ the equation (6) reduces to

$$\log P = -\tfrac{1}{2}\log \varrho_t - 0.936 \tag{8}$$

and (8) indicates the period of a small object which rotates around the star at a distance from the centre almost equal to the radius.

In figure 1 the equation (8) is indicated by a dotted curve. The above suggests, that the true value of the coefficients A and B in the relations 2, 3 and 4 is exactly 0.5 and that the small deviations from this value are entirely accidental e. g. due to the choice of the pairs which are used in the determination of the coefficients A and B. At the same time the coefficients B will be about equal to the mean difference

$$\left\{\frac{3}{2}\overline{\log\left(1 + \frac{r_d}{r_t} + \frac{s}{r_t}\right)} - \frac{1}{2}\overline{\log\left(1 + \frac{\mathfrak{M}_d}{\mathfrak{M}_t}\right)}\right\} - 0.936$$

while the probable error of the deviations $\varDelta = \log P_0/P_c$ corresponds to the scatter of the individual values

$$\left\{\frac{3}{2}\log\left(1 + \frac{r_d}{r_t} + \frac{s}{r_t}\right) - \frac{1}{2}\log\left(1 + \frac{\mathfrak{M}_d}{\mathfrak{M}_t}\right)\right\}.$$

Solution with $A = 0.5$.

If for the three groups of binaries for A the value $A = 0.50$ is adopted, the corresponding value of B simply is equal to

$$B = \overline{\log P} - \tfrac{1}{2}\overline{\log \varrho_t} .$$

Instead of 2, 3 and 4 we then find

$$\left.\begin{array}{lll} \log P = -0.5 \log \varrho_t - 0.224 & r = \pm 0.151 & n = 31 \\ \log P = -0.5 \log \varrho_t - 0.213 & r = \pm 0.208 & n = 273 \\ \log P = -0.5 \log \varrho_t - 0.238 & r = \pm 0.247 & n = 82 \end{array}\right\} \quad (9)$$

As appears from the values r, the distribution of the individual points is almost equally well represented by the relations 9 as by the relations 2, 3 and 4.

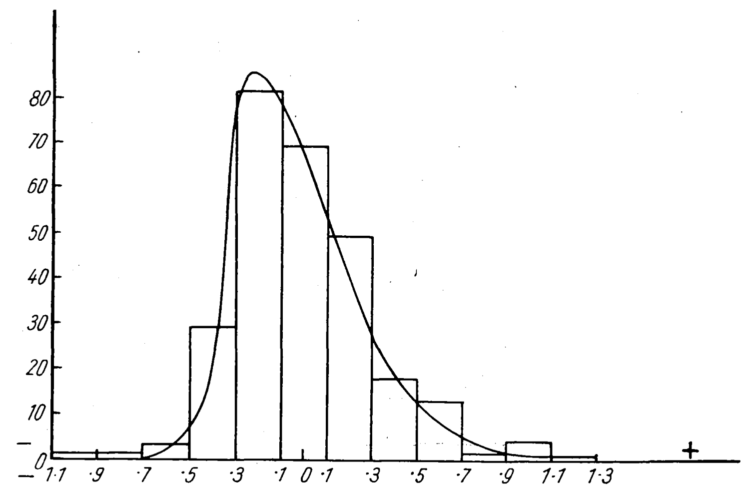

Fig. 2. Distribution of the values $\Delta = \log P_{obs} - \log P_{comp}$ (see table 2). The distribution of the values Δ is a non symmetrical one.

The value of B is around —0.225. We may therefore conclude that for the spectroscopic binaries and for the eclipsing binaries the mean value

$$\frac{3}{2} \overline{\log\left(1 + \frac{r_d}{r_t} + \frac{s}{r_t}\right)} - \frac{1}{2} \overline{\log\left(1 + \frac{\mathfrak{M}_d}{\mathfrak{M}_t}\right)} = +0.711 \quad (10)$$

while this mean does not systematically depend on period or density. For three groups of binaries considered here, the scatter of the individual the values (10) are ± 0.151; ± 0.208 and ± 0.247 respectively.

There is no upper limit for positive deviations $\Delta = \log P_0/P_c$ but the relation (8) sets a limit to negative values. This limit will only be transgressed by pairs for which

$$\frac{\mathfrak{M}_t}{r_t^3} < \frac{\mathfrak{M}_t + \mathfrak{M}_d}{(r_t + r_d)^3}$$

that is for pairs in which the dense component $\mathfrak{M}_d$ is very massive and to which special attention should be given (see below).

Consequently the distribution curve of the values Δ can be expected to be asymmetrical and this is what appears from table 1 and from fig. 2 in which the numbers of table 1 are graphically represented.

Table 1. *Distribution of the values* $\Delta = \log P_0 - \log P_c$.

$\Delta =$	+1.3	+1.1	+0.9	+0.7	+0.5	+0.3	+0.1	−0.1
n	1	4	2	14	17	49	69	
$\Delta =$	−1.3	−1.1	−0.9	−0.7	−0.5	−0.3	−0.1	
n	0	1	1	3	30	82		

The Systems V Crt and TU Mus.

As appears from the table and from figure 1, two systems have a period definitely below the limit set by 8. These two pairs are V Crt and TU Mus for which the following data are enumerated by GAPOSCHKIN.

V Crt $P = 0^d.702$; Sp (1) = A6; $M_{1\,abs} = 2.5$; $\mathfrak{M}_1 = 2.3$; $R_1 = 2.1$
Sp (2) = (F5); $M_{2\,abs} = 3.0$; $\mathfrak{M}_2 = 1.2$; $R_2 = 4.2$

TU Mus $\varrho = 1^d.387$ Sp (1) = B3; $M_{1\,abs} = -2.2$; $\mathfrak{M}_1 = 8.1$; $R_1 = 14.0$
Sp (2) = B3; $M_{2\,abs} = -2.1$; $\mathfrak{M}_2 = 7.9$; $R_2 = 13.6$

For V Crt from KEPLER's third law (5) the semi axis major of the system is found to be

$$a = \{74.3\ (\mathfrak{M}_t + \mathfrak{M}_d)\ P^2\}^{1/3} = 5.02\ (R_\odot = 1).$$

while $r_1 + r_2 = 6.3$.

In his catalogue GAPOSCHKIN [3] also gives the mean light curve and in this both the primary and secondary minimum are well indicated. The present values however lead to results which are controversial.

This is still more the case with TU Mus. For the semi axis major KEPLER's third law gives $a = 13.2$ ($r_\odot = 1$) while $r_1 + r_2 = 27.6$. The light curve given by GAPOSCHKIN gives two minima of almost equal depth. It would seem that either a different period must be assigned to this system or that the star is not an eclipsing variable. Anyhow, with this variable also the present values, basedon photometric data, lead to controversial results.

The Periods of Axial Rotation of Single Stars.

Table 2 gives the velocities of axial rotation (in km/sec) for stars of different spectral types as derived by CHANDRASEKHAR, MÜNCH and by STRUVE [1]. In this same table are given the values of the radius $R/R_\odot$ and the density $\varrho/\varrho_\odot$ corresponding to these types as given by GAPOSCHKIN [2].

From the values of V and R we easily find the corresponding values of log P as given in the table (P in days). If these periods are represented by an equation of the form (1) for the separate spectral types, the values B, as given in the final column of the table, are obtained.

Table 2. *Rotational velocities and periods of axial rotation of the stars of different spectral type. Values of the coefficient B in* $\log P = -0{,}5 \log\varrho + B$.

Spec.	V km/sec	$R/R_\odot$	$\varrho/\varrho_\odot$	$\log P$	B
Oe—Be	350	30 :	10^{-3}:	+0.638	—0.862
O—B	94	5.8	0.04	+ .495	—0.204
A	112	1.64	0.41	— .129	—0.323
F0—F2	51	1.40	0.52	+ .144	+0.002
F5—G5	20	1.23	0.62	+ .494	+0.390
dG — dM	0	—	—	∞	∞

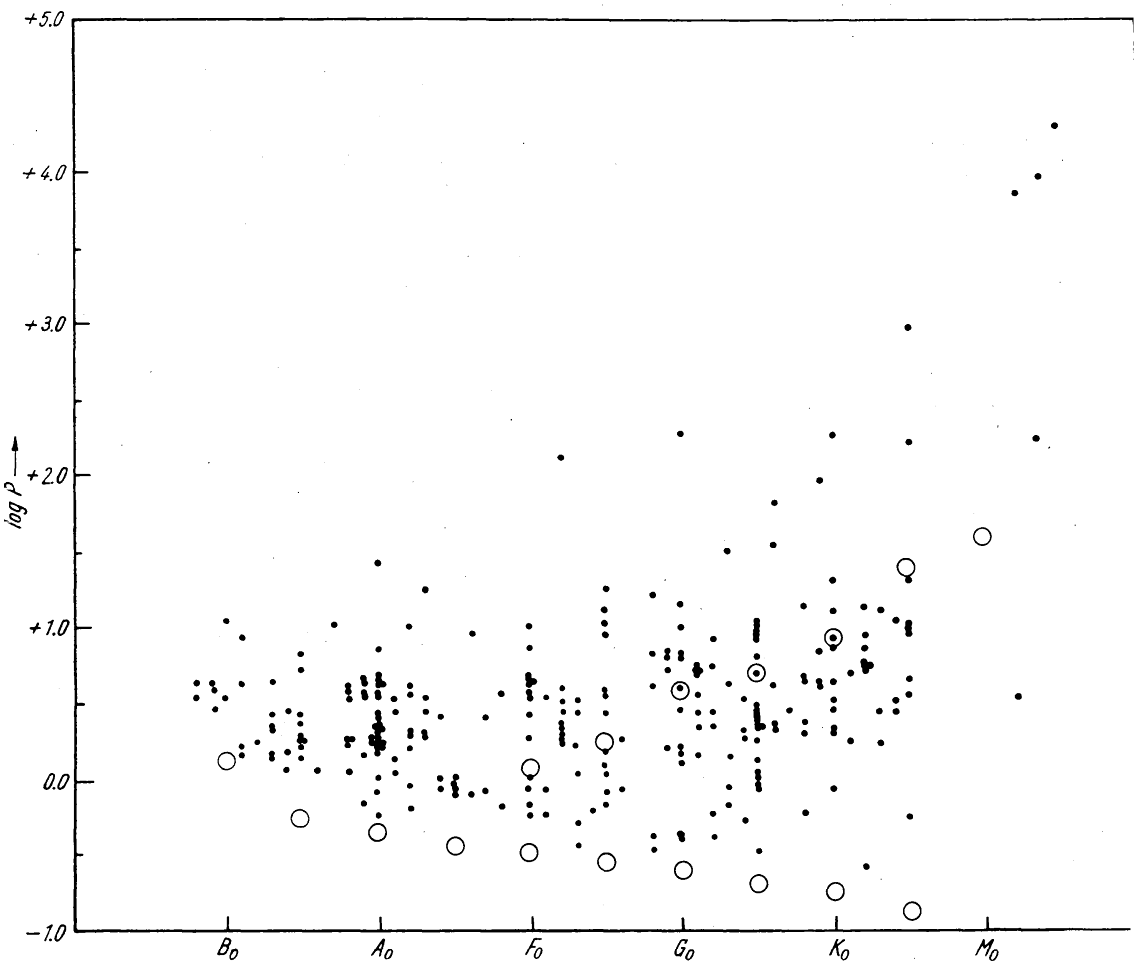

Fig. 3. Relation of logarithm of period and spectral type of tenuous component. Larger circles indicate the minimum periods of revolution of pairs consisting of two equal components, each component either belonging to the main or to the giant series.

In figure 1 the values $\log P$ from table 2 are indicated by large open squares. For the early type stars the periods of rotation of the single stars seem to be of the same order of magnitude as the periods of the binaries. For the Oe — Be stars the period of axial rotation may be smaller, but for these stars the values of $R/R_\odot$ and $\varrho/\varrho_\odot$ are rather uncertain, and not much weight can be attached to the corresponding values $\log P$ and B. With the stars later than F2, no large rotational velocities have been observed. This does not necessarily mean that for these stars the rotational velocities are equal to zero, because below a certain value rotational velocities are difficult to measure.

It does indicate however that for these later stars, in whatever way the absence of high rotational velocities is explained, the periods are not of the same order of magnitude as the periods of revolution of the binaries of the same spectral type.

Period and Spectral Type.

In fig. 3 for the stars in GAPOSCHKIN's list the logarithms of the periods are plotted against the corresponding spectral types of the tenuous component. The large open discs were obtained in the way described by MARTINOFF [5]. The discs represent the period of two stars with dimensions normal for the spectral type, which rotate around each other while they are in contact. Both for the main sequence stars and for the giants, the dimensions were taken from the same paper by GAPOSCHKIN as those used in table 2. From this diagram the points, corresponding to V Crt and TU Mus, have been omitted. In the diagram the main series and the giant series are not clearly distinguished.

The Masses.

With the spectroscopic pairs [4] there was a notable absence of stars with masses between the limits $\log \mathfrak{M} = 0.4$ and 0.6 and as a result these stars seemed to fall apart into two groups. With the more extensive material of GAPOSCHKIN, this falling apart into two groups has completely disappeared. As all data concerning the masses are given in GAPOSCHKIN's original paper [3] the masses are not tabulated here.

Literature.

[1] CHANDRASEKHAR, S., G. MÜNCH and O. STRUVE: Stellar Evolution **1949**, 151. — [2] GAPOSCHKIN, S.: J. R. Astr. Soc. Canada **33**, 239 (1939); [3] H. A. **113**, 2 (1953). — [4] KREIKEN, E. A.: Z. Astrophysik **31**, 256 (1952). — [5] MARTINOFF, A. G.: Astron. Z. J. **14**, 306 (1937). — [6] MOORE, J. H., and F. J. NEUGEBAUER: L. O. B. **20**, 1 (1948). — [7] PLAUT, L.: Gron. Publ. **55**, 28 (1953).

Prof. E. A. KREIKEN, Department of Astronomy, University of Ankara.