CLOSE BINARY SYSTEMS BEFORE AND AFTER MASS TRANSFER: A COMPARISON OF OBSERVATIONS AND THEORY*

J. P. DE GRÈVE** and D. VANBEVEREN
Astrophysical Institute, Vrije Universiteit Brussel, Brussels, Belgium

(Received 9 August, 1979)

Abstract. From a search through the literature 174 close binaries with known absolute dimensions have been sampled. Distinction is made between systems before and after mass exchange. Mass, period and mass ratio distributions and relations of the group of 'unevolved' binaries (i.e., prior to mass exchange) are transformed into corresponding distributions and relations of evolved binaries. The transformations are based upon the $M_{1f} = g(M_{1i})$ relation derived from an extended set of published theoretical computations of the evolution of close binaries. From this relation the following characteristics of the system after mass exchange are computed: M_{1f}, M_{2f} (and q_f), P_f. Five different modes of mass transfer were applied for the computation of the values of P_f and M_{2f}. The variation of the period was calculated using the formalism given by Vanbeveren *et al.* (1979). The results are compared to the observations of binary systems after mass exchange, and are discussed together with an analysis of the effect of several selection effects present in the distributions. The main conclusion is that, during mass exchange in close binaries, more than 50% of the mass is lost to the system in the process of transfer, removing a large amount of angular momentum.

1. Introduction

Evolutionary computations on close binary evolution involving mass exchange were developed in the 1960s after the scenario became evident from observational and theoretical aspects. It has since then been a major tool in understanding the status of several classes of close binary systems as well as in making a link between those classes (see the reviews of Paczynski, 1971; Plavec, 1973; van den Heuvel, 1976; Thomas, 1977 and references therein). Most of the theoretical work was concentrated on conservative mass exchange. In almost all the papers the results of the evolution are compared with observed systems. Often the comparison converges towards small groups of stars or individual systems. Some classes of stars are linked to different stages of theoretical evolution, such as WR systems and X-ray binaries.

In this paper we compare the characteristics of two general groups of stars, using a basic property of theoretical binary evolution that the final mass of the primary depends nearly on the initial mass only. Through the transformation of a first group of systems into a group comparable to the second, conclusions are derived on the

* This research is supported by the National Foundation of Collective Fundamental Research of Belgium (F.K.F.O.) under No. 2.9009.79.
** Research Fellow, NFWO, Belgium.

Astrophysics and Space Science **68** (1980) 433–457. 0004–640X/80/0682–0433$03.75

mass loss from the system as well as on the angular momentum the lost mass takes with it. In Section 2 the relevant theoretical transformation formulae are given. In Section 3 the two groups of systems are defined, constructed and analysed. Section 4 deals with the comparison of the transformed systems to the observed. Conclusions are given in Section 5, together with a discussion.

2. Theoretical Evolution: Formalism

A search through the literature, starting from Webbink's (1975) table of theoretical computations on close binary evolution, resulted in a fairly complete list of 151 results on theoretical mass exchange computations. The characteristics of this list are:

(a) A large choice in initial parameters; i.e.,

$$1\ M_\odot \leqslant M_{1i} \leqslant 64\ M_\odot$$
$$0.2 \leqslant q_i \leqslant 0.94$$
$$0.544\ \text{d} \leqslant P \leqslant 216\ \text{d}$$

(b) The list contains 105 conservative calculations; the non-conservative cases also contain some studies including stellar wind mass loss.

(c) Among the conservative cases, twenty-two case A studies and fifty-six case B studies are found.

(d) A spread in average mass loss rate and mass exchange time-scale occurs for the parameters of the same order of magnitude. This is mostly due to small differences in the computer programs, techniques and input physics, used by different authors.

From all these studies we will restrain the following properties, already mentioned by other authors:

(a) The final mass of the primary after the mass exchange M_{1f} is nearly independent of the initial values of P and q (at least for case B).

(b) M_{1f} is nearly independent of mass or angular momentum lost from the system.

In the following we will give the relations for the system parameters M_1, period P and $q = M_2/M_1$; that will enable us to transform each unevolved system into a system after mass exchange. For the transformation of P and q, three different cases are considered:

C: Conservative case (conservation of mass and angular momentum).

NC50: Non-conservative case with 50% of the transferred mass lost from the system.

NC100: Non-conservative case with all the transferred mass lost from the system.

2.1. Remnant mass of the primary

As was already outlined in a review paper by Plavec (1968), the behaviour of the mass exchange in case B is different for the different masses of the primary, the key values being 2.8 $M_\odot$ and 9 $M_\odot$. We therefore derived three different relations for M_{1f} from the case B data.

To derive consistent transformation formulae for M_{1i}, we restrain only the conservative case A and case B computations in the list. Models with the same value of M_{1i} were averaged over the final value M_{1f} (so each value M_{1i} in the list was given the same weight). Using a best fit curve program, the following relations were found:

Case A $\quad M_{1f} = M_{1i}/(2.16094 + 4.9004E-2\ M_{1i})$ (1)
with maximum error $P_m = 20\%$

Case B1 $\quad M_{1f} = 0.297727 + 1.19871E-2\ M_{1i}$ (2a)
with $P_m = 17\%$ and $1\ M_\odot \leqslant M_{1i} < 2.79\ M_\odot$

Case B2 $\quad M_{1f} = M_{1i}/(10.6264 - 0.785497\ M_{1i})$ (2b)
with $P_m = 36\%$ and $2.79\ M_\odot \leqslant M_{1i} < 6.265\ M_\odot$

Case B3 $\quad M_{1f} = M_{1i}^{1.48945} \times 7.13935E-2$ (2c)
with $P_m = 36\%$ and $6.265\ M_\odot \leqslant M_{1i}$

The large errors result from the non-homogeneity of the set. The boundaries in Equations (2a)–(2c) result from the assumption of continuously increasing values of M_{1f} for increasing M_{1i} values. We derived no relation for case C as only two papers deal with this case and only a few observed systems were found as candidates for this kind of mass exchange.

2.2. The mass ratio q

The mass ratio after mass exchange is given by

$$q_f = M_{2f}/M_{1f} \quad \text{with } M_{2f} = M_{2i} + \beta(M_{1i} - M_{1f}), \tag{3}$$

where $\beta = 1$, 0.5, 0 for the respective cases C, NC50 and NC100.

2.3. The period P

For the period change due to loss of orbital angular momentum the equation derived by Vanbeveren *et al.* (1979) is used:

$$P_f = P_i \left(\frac{M_{1f} + M_{2f}}{M_{1i} + M_{2i}}\right)^{3\alpha + 1} \left(\frac{M_{1i}M_{2i}}{M_{1f}M_{2f}}\right)^3 \tag{4}$$

with α ($\geqslant 0$) defined by

$$c_\alpha = \frac{\Delta J}{J}(\alpha) = 1 - \left(1 - \frac{\Delta M}{M_{1i} + M_{2i}}\right)^\alpha . \tag{5}$$

Vanbeveren *et al.* expect values of α in the range 0–4. For the sake of simplicity we choose α equal to 1 and 3, corresponding to a small and large amount of angular momentum leaving the system. (This leads to a corresponding notation NC51, NC53 and NC101, NC103.) The exact amount of angular momentum for each system depends on the complete set of parameters. To give an idea of that amount we

TABLE I

Influence of parameter α on the angular momentum loss, for $\alpha = 1$ and 3; $c_\alpha = (\Delta J/J)(\alpha)$. All M-values are given in $M_\odot$

System	M_{1f}	ΔM(NC50)	c_1(NC50)	c_3(NC50)	ΔM(NC100)	c_1(NC100)	c_3(NC100)
30 + 27	11.32	9.34	0.16	0.42	18.68	0.32	0.70
30 + 9	11.32	9.34	0.24	0.56	18.68	0.48	0.86
3 + 2.7	0.33	1.33	0.23	0.55	2.67	0.47	0.85
3 + 0.9	0.33	1.33	0.34	0.71	2.67	0.68	8.97

computed the angular momentum loss for some examples, with remnants M_{1f} given by Equations (2). The results are shown in Table I.

3. Observations: Unevolved and Evolved Close Binaries

3.1. Definition

We define two groups of binary systems:

(a) *Unevolved systems* (set I): Systems that have not been influenced by mass exchange (i.e., both components still evolve as single stars).

(b) *Evolved systems* (set II): Systems that have undergone mass exchange or systems that are nearly at the end of that stage (i.e., Algol-type systems).

The division between sets I and II is made by comparing the data of the two components with evolutionary tracks of single stars in the HRD (with initial composition $X = 0.70$, $Z = 0.03$). Moreover, the radii of both stars are compared to their respective critical Roche radii. When the comparison gave no definite conclusion, or when the data lead to contradictory results (for example, the data of HD 190967 result in radii of the components that are two to three times larger than the Roche radii), the system is classified as 'undefined'.

From a search through the literature, systems were gathered with known absolute dimensions. All the data are brought into the same form, namely M_i, L_i, $T_{e,i}$, R_i ($i = 1, 2$). Where necessary effective temperatures and bolometric corrections were derived from the spectral type, using the values given by Lang (1974) – see also Aitken (1964) and Flower (1977). When different spectral subclasses are given by different authors, the result of the most recent paper is taken if the latter contains more extended observations than earlier papers. The Roche radii are computed with the usual relations (Paczynski, 1966; Horn *et al.*, 1969). The search gave a list of 174 systems, which are given in the Appendix together with references.

After a computer-guided division into the three sets (I, II and 'undefined') all systems were checked a second time, interpreting the data in the HR diagram. The final result is a group of 100 unevolved systems and a group of 40 evolved systems. Although these samples are still statistically small and, through the demand of completeness of data, subject to some severe selection effects (double-lined and eclipsing binaries are favoured, as well as massive systems, the latter through their large velocity amplitudes and/or high luminosity), we consider sets I and II to be sufficiently representative to transform one group into a group comparable to the other. Remarks and discussion upon the selection effects in the distributions of the parameters are given in Sections 3.2 and 4. We emphasize that no corrections were applied to the sets to take these effects into account.

3.2. General characteristics of the observations

The histograms of log M_1, q and log P are given in Figures 1, 3 and 4 for set I, in Figures 7, 8 and 9 for set II. The bin width of the histograms is, respectively, 0.2, 0.1 and 0.4.

(a) *The* log M_1 *distributions*

The log M_1 distribution of set I has a maximum in the mass interval (1.58 $M_\odot$, 4 $M_\odot$). The fact that we consider systems without any restriction on their space distribution induces the selection effect that more massive stars are overabundant because they are intrinsically brighter. In order to have an idea of the influence of this effect, we first restricted ourselves to stars within a distance of 20 pc, published in the catalogue of Batten (1970) with complementary catalogues of Pédoussant and Ginestet (1971),

TABLE II

Limiting visual magnitudes for different distances for stars with spectral types ranging from G2 to B2

Spectral type	Limiting visual magnitude for a distance $D \leqslant 20$ pc.	Limiting visual magnitude for a distance $D \leqslant 50$ pc.	Limiting visual magnitude for a distance $D \leqslant 137$ pc.
G2	6.5	8.5	
F8	5.75	7.75	
F5	5.5	7.5	
F2	4.81	6.81	
F0	4.9	5.9	
A7	4.0	6.0	
A5	3.65	5.65	
A3	3.13	5.13	
A2	3.13	5.13	
A1	3.23	5.23	
A0	3.33	5.33	
B9	2.32	4.32	6.5
B8	1.96	3.96	6.14
B7	1.68	3.68	5.86
B6	1.54	3.54	5.72
B5	1.23	3.23	5.41
B3	0.65	2.65	4.83
B2	0.49	2.49	4.67

TABLE III

The correspondence between spectral type and mass for luminosity class V and VI stars. Evolutionary calculations are used for Population I stars with masses ranging from 1 $M_{\odot}$ up to 10 $M_{\odot}$

Spectral type	Mass range ($M_{\odot}$)
G2 → F3	|1, 1.5|
F2, F1	|1.4, 1.6|
F0 → A7	|1.5, 2|
A6, A5	|1.8, 2.3|
A3 → A0	|2. 3|
B9, B8	|3, 4|
B7 → B5	|4, 6|
B3, B2	|6, 10|

Pédoussant and Carquillat (1973), and Pédoussant and Nadal (1977). If we know the spectral type, luminosity class and visual magnitude, this restriction can easily be done. We omitted the luminosity class III, II and I stars as *they may be evolved stars*. For a number of stars no luminosity class is available. As class V, IV correspond to hydrogen core burning and class III, II and I correspond to hydrogen shell burning (and some of them with He burning), class V, IV stars should appear much more frequently (> 10 times more). Therefore, if no luminosity class is available, we assume class V. The restriction on distance induces a restriction on the visual magnitude for every spectral type. This is shown in Table II. Assuming that the catalogue is fairly complete up to visual magnitude 6.5, we only consider stars of spectral type G2 and earlier (i.e., $M \geqslant 1\ M_{\odot}$). Every spectral type then corresponds to a certain mass range. In Table III the correspondence between spectral type and mass range is given using evolutionary computations for Population I stars with masses of 1 $M_{\odot}$ up to 20 $M_{\odot}$ (de Grève, 1979, unpublished). It is obvious that, with such a procedure, we do not expect to see a large number of B type stars for example. Having an indication of the difference between the first two mass intervals considered here, we can increase our distance up to 50 pc in order to know something about the difference between the following mass intervals. We continued that system until we obtained the distribution given in Figure 1. The distribution of set I and the distance distribution were normalized by equalling the number of systems for both of them in the mass intervals considered. From the figure it is obvious that the selection effect is largest in the interval $|1\ M_{\odot}, 1.6\ M_{\odot}|$, as was expected. Figures 1b and 1c, however, show that the distribution resulting from our selected sample of stars gives a fair representation of reality in the mass range $|1.6\ M_{\odot}, 15\ M_{\odot}|$, the low mass stars being slightly underabundant. An interesting feature also results from a comparison with the mass function of Lequeux (1979) who did not separate binaries and single stars. From the

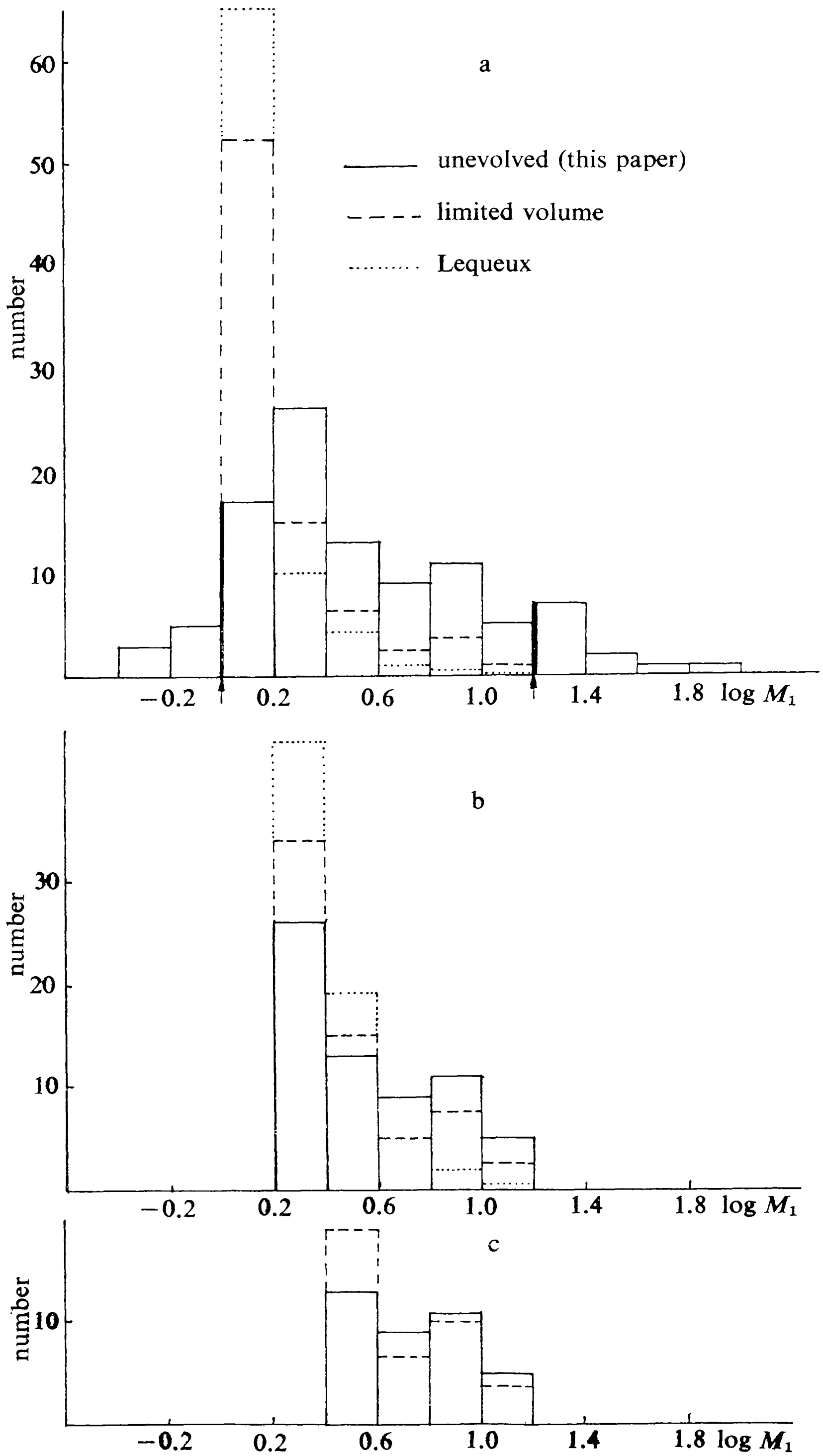

Fig. 1. The mass distribution for the set of unevolved stars (full line) and for a set of stars within a limited volume (dashed line). The dotted line shows the mass distribution for binaries and single stars determined by Lequeux (1979). (a) The three distributions are normalized to the total number of stars in the mass range $0.0 \leqslant \log M \leqslant 1.2$. (b) The normalization is performed to the number of stars in the mass range $0.2 \leqslant \log M \leqslant 1.2$. (c) The normalization is performed to the number of stars in the mass range $0.4 \leqslant \log M \leqslant 1.2$.

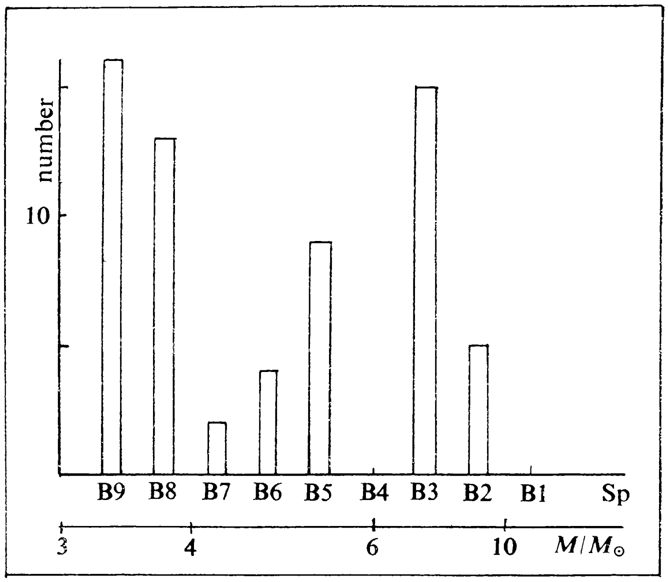

Fig. 2. The distribution of B V, B IV type stars within a distance of 137 pc.

comparison, there seems to be a difference between the mass distribution in single stars and in spectroscopic binary systems. Moreover, the increase of the number of stars in the interval $|6\ M_{\odot}, 10\ M_{\odot}|$ may be real. This can be seen by considering the spectral type distribution for V and IV stars (i.e., the mass range $3\ M_{\odot} \rightarrow 10\ M_{\odot}$ is considered). The results are shown in Figure 2. They point clearly towards a bimodal distribution for the masses of primaries of spectroscopic binaries, with the secondary maximum in the interval $|6\ M_{\odot}, 10\ M_{\odot}|$. An investigation on the nature or origin of this bimodality could be quite interesting, but goes beyond the scope of this paper.

The log M_1 distribution for evolved systems (set II), shown in Figure 7, has a maximum in the mass interval $|0.4\ M_{\odot}, 0.63\ M_{\odot}|$. Again, of course, the same selection effect is present as for non-evolved systems – i.e., low mass stars are fainter and thus not as extensively studied. Now we can ask ourselves if part of this distribution may be representing the real distribution. A remnant star after Roche lobe overflow has almost the same luminosity that the primary star had just before Roche lobe overflow (see, for example, Ziolkowski, 1970; de Grève *et al.*, 1978), even if we include stellar wind and external mass loss (Vanbeveren *et al.*, 1979). It is therefore not unreasonable to state that evolved stars are observable with the same probability as their corresponding non-evolved precessors (but, of course, in smaller numbers). For the non-evolved stars in the mass range $|1.6\ M_{\odot}, 15\ M_{\odot}|$ the log M_1 distribution is real enough if we consider the fact that we are working with small numbers. Using the formalism given in Section 2, we conclude that for the evolved stars in the mass range $|0.4\ M_{\odot}, 2.5\ M_{\odot}|$ the distribution is also close to reality, again with the low mass stars being slightly underabundant.

(b) *The q distribution*

We will restrict ourselves here to the distribution of set I. The distribution of set II will be considered in Section 4. The mass ratio histogram (Figure 3) shows a double peak, the first situated in the range $q = |0.3, 0.5|$, the second in the bin $|0.9, 1.0|$. But the first peak is only about one-third of the second, a feature completely different from Trimble's distribution (1974, 1978). In her distribution based upon the systems in the catalogue of Batten (1970) and the supplements to this catalogue (Pédoussant and Ginestet, 1971; Pédoussant and Carquillat, 1973; Pédoussant and Nadal, 1977), the first peak occurs in the bin $|0.2, 0.3|$ and is slightly larger than the second peak, both being of the order of 15%. The lack of low mass ratios in our sample is due to the demand of completeness of the data which favours largely the double-lined spectroscopic binaries ($q \sim 1$). The influence of this lack on the results of the transformation is discussed in Section 4.

(c) *The P distribution*

The $\log P$ distribution of the unevolved systems is shown in Figure 4, that of the evolved in Figure 9. The overabundance of periods around three days is obvious, as a result of selection effects. This feature is present in the first set and also in the second. Unfortunately, no extended statistical studies of the separate groups exist. More global studies, such as those undertaken by Abt and Levy (1976, 1978; see also references therein), demonstrate the existence of a peak in the P distribution (~ 14 yr) and a more or less homogeneous distribution in the range $1–10^6$ days (B2–B5 and F3–G5 IV or V stars were considered). The influence of the lack of large period systems in both samples is discussed in Section 4.

(d) *Different modes of mass transfer*

Knowing the period of the systems, one can decide upon the case of mass transfer (A, B or C), as was outlined by Plavec (Plavec, 1968; Iben, 1967) for masses up to

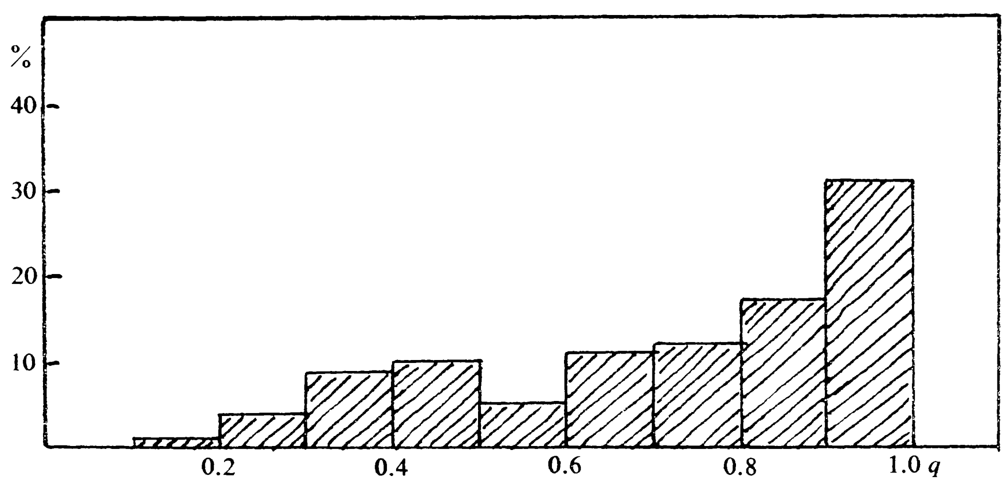

Fig. 3. Mass ratio distribution of unevolved close binaries (set I).

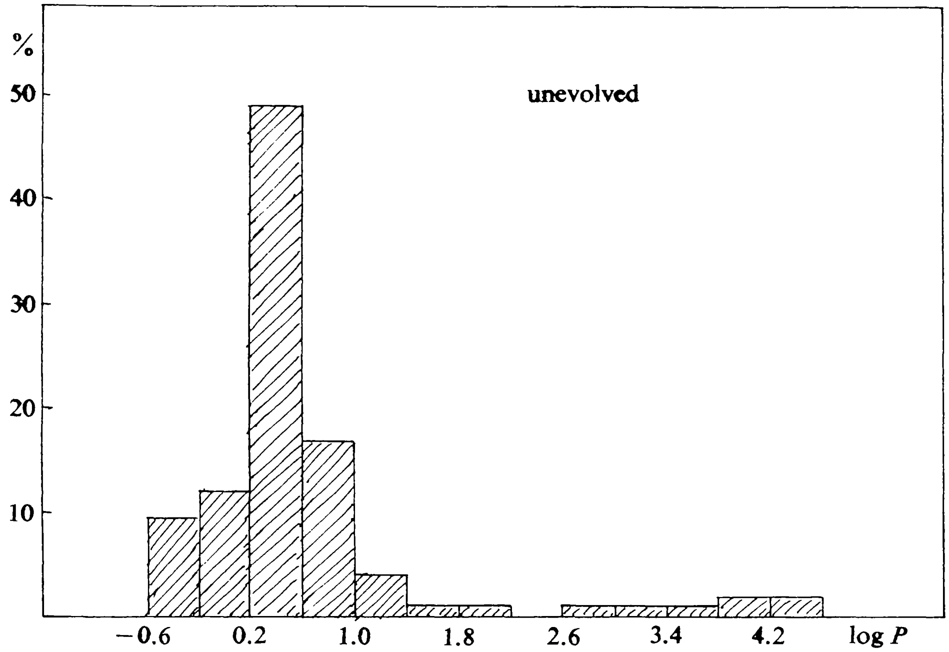

Fig. 4. Period distribution of unevolved close binaries (set I).

15 $M_{\odot}$ and the values and formulae given by de Grève (1976) for masses up to 30 $M_{\odot}$, the 100 unevolved systems were divided into three subclasses: sixteen case A systems, seventy-eight case B systems, six case C systems. This means that in our sample the number of case A systems relative to the number of case B systems amounts to ~20%.

It is interesting to note the peculiar feature that the fraction of case B systems (relative to the number of (case A + case B) systems) increases with the number of observed systems. De Grève (1976), using fifteen massive binaries, found 67% case B systems, Plavec (1968), using forty-five binaries (with maximum brighter than $8^{m}_{\cdot}5$), found some 76%, while the present study of ninety-four A and B systems gives 83%. This is not so surprising if we consider the evolutionary arguments. As was discussed earlier (Plavec, 1968; Paczynski, 1971) the mode of mass exchange is determined by the value of the Roche radius versus the radii R' and R'' (respectively, the maximum values of radius during hydrogen core burning and hydrogen shell burning). That value is determined by the period of the system. If we denote by $p_{\rm A}$ (resp. $p_{\rm B}$) the probability for a system to have mass transfer following case A (resp. case B), and if we assume that the periods are more or less equally divided over all possible values (in fact the only restriction is that no overabundance occurs in the range $0 < P <$ ~2 d) then $p_{\rm A}$ is given by the probability that the period is such that $R_{\rm ZAMS} \leqslant R_R < R'$ (resp. $R' \leqslant R_R < R''$ for $p_{\rm B}$), or

$$\frac{p_{\rm B}}{p_{\rm A}} = \left(\frac{R' - R''}{R_{\rm ZAMS} - R'}\right)^{3/2}$$

with $R_{\rm ZAMS}$, the value of the radius on the ZAMS.

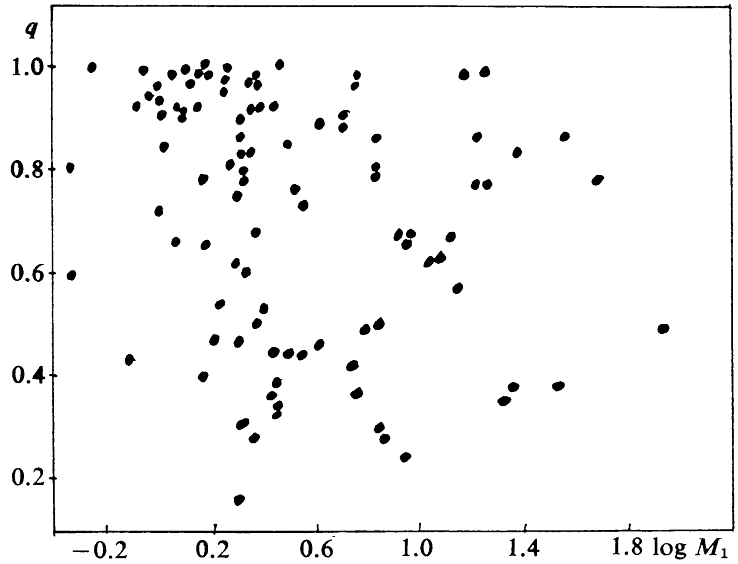

Fig. 5. q–log M_1 diagram for the binaries of set I (unevolved).

Simple calculations show that this probability varies from ~94% for $M = 2\ M_\odot$ to ~99% for $M = 30\ M_\odot$. It is obvious that if the period distribution is not homogeneous but concentrated towards larger values ($P > 2$–3 days), then the former is still valid. However, the influence of the case C systems must then be taken into account. A last remark in this respect is that for massive binaries ($M_{1i} > 15\ M_\odot$) stellar wind mass loss of both components influences the period, which results in a favouring of case B systems (Vanbeveren *et al.*, 1979; Vanbeveren and de Grève, 1979).

(e) *Correlation of P and q with M*

In order to find out whether or not a correlation exists between P and M or q and M, we plotted both P and q as a function of the mass M_{1i}. The existence of such correlations is important in case only a fraction of the systems can be transformed. The results, shown in Figures 4 and 5, demonstrate that no clear relationship exists between those parameters – i.e., each mass bin is occupied by systems with both high and low values of P and q.

(f) *The set of transformed systems* (II′)

As mentioned earlier, we exclude the case C transformation as well as the observed case C systems. From the latter six are present in set I. With the equations given in Section 2 the resulting ninety-four unevolved systems are transformed into systems after mass exchange. This set of transformed systems is denoted by II′, and in the following section compared to set II.

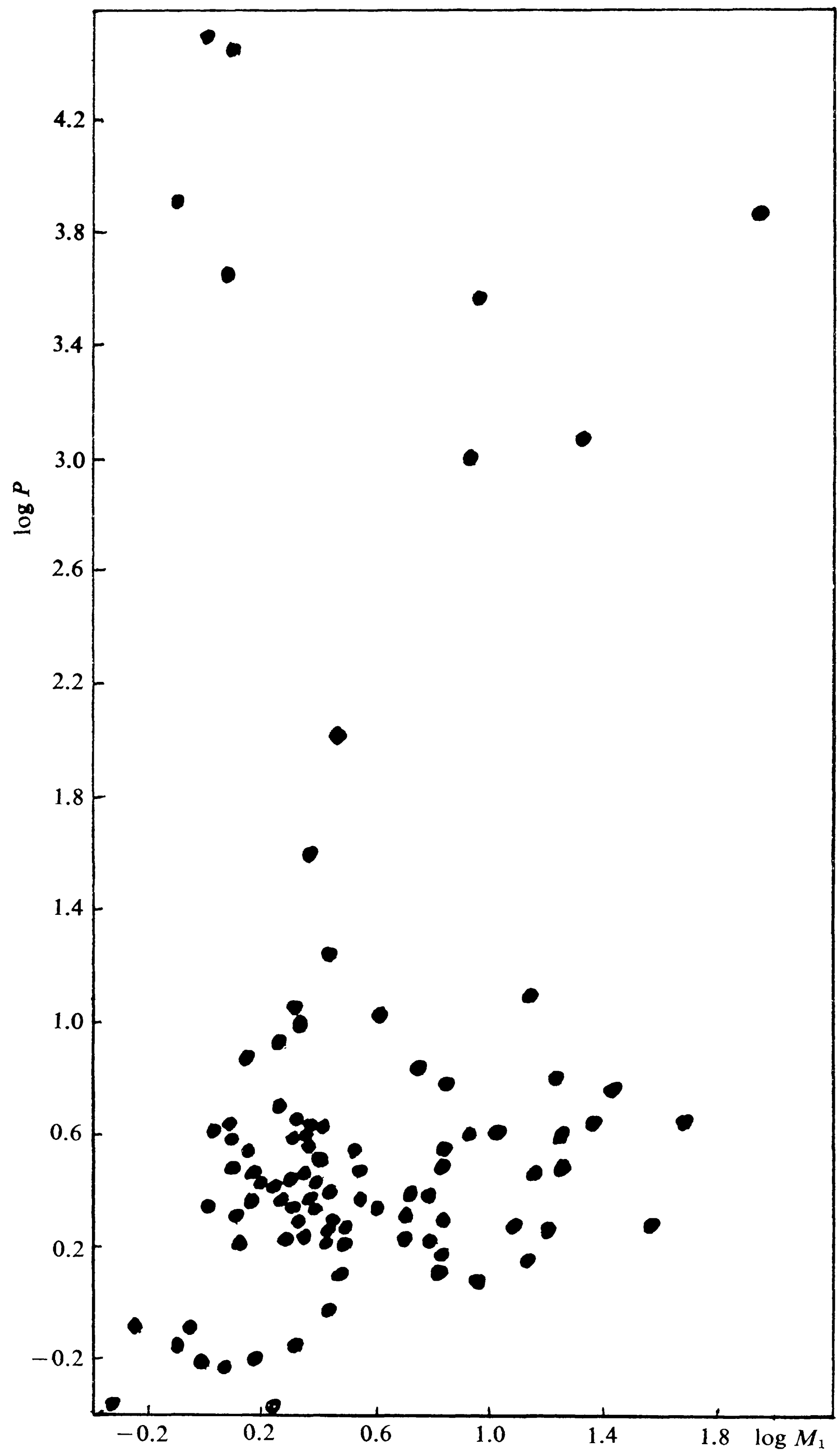

Fig. 6. log M_1–log P diagram for the binaries of set I (unevolved).

4. Comparison between observations and evolutionary calculations

4.1. The mass distribution

In order to get an idea of the validity of the theoretical formalism presented in Section 3, we can transform the non-evolved systems with the aid of formulae 4 and 5a, b, c. The distribution of the transformed systems then has to be compared with the observed histogram for evolved stars (Figure 6). However, four effects must be taken into account:

(a) Stars from set I with the same mass were not born at the same time. As the birthrate function is time dependent, stars in our sample with the same mass can be formed at different rates. Therefore, transformation of all these systems would give different results compared to the observed evolved systems (due to the shorter He burning lifetime holding for evolved stars we may reasonably expect that all the systems of set II are formed at the same rate). We can get around the problem by restricting ourselves to stars with masses higher than 3 $M_\odot$ (with a time-scale of hydrogen core burning much lower than the typical time-scale for the variation of the birthrate). For stars with masses larger than 3 $M_\odot$ we also avoid the selection effect present in the low mass range (see Section 2).

(b) Stars with different masses have different hydrogen Main-Sequence lifetimes.

(c) Evolved stars are mostly in a stage of He burning. This stage is much shorter than the hydrogen Main-Sequence lifetime holding for non-evolved stars. On the other hand, He burning stars with different masses also have different lifetimes.

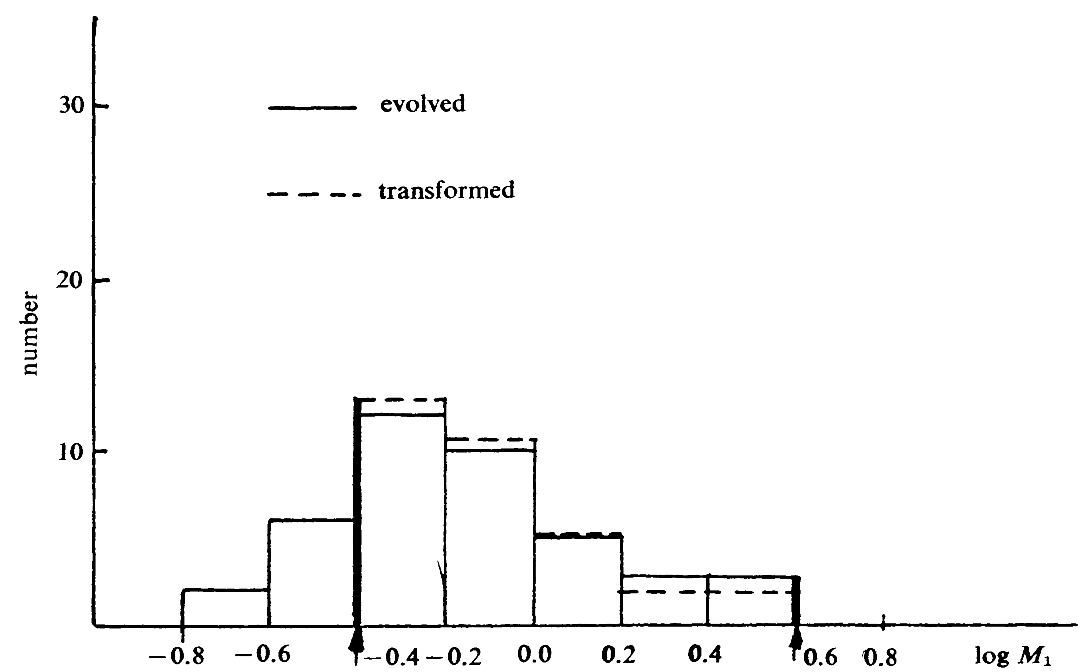

Fig. 7. Distribution of M_1 for set II (evolved; full line) and for the transformed systems (set II′; dashed line). The normalization is performed with respect to the total of stars in the mass range $-0.4 \leqslant \log M \leqslant 0.6$.

(d) Stars with masses larger than 15 $M_\odot$ may lose mass to the stellar wind. They evolve in an alternative way. For more detailed information about the behaviour of binaries in this mass range we refer to Vanbeveren *et al.* (1979), Vanbeveren and de Grève (1979), Vanbeveren and de Loore (1979). We will restrict ourselves to stars with masses lower than 15 $M_\odot$.

We proceed as follows: the mass intervals of the observed distribution of evolved stars are transformed into mass intervals of the non-evolved processors by means of formulae 5 and 6. The evolved stars in these mass intervals are counted. These counts are then divided by the average hydrogen Main-Sequence lifetime holding for the mass interval considered, and multiplied by the He burning lifetime holding for the corresponding evolved mass interval. Normalization is performed by equalling the total number of stars for the transformed and the observed distribution of evolved stars in the range $-0.4 \leqslant \log M_1 \leqslant 0.6$. The results, plotted in Figure 7, show a striking correspondence between theory and observation, indicating that the theoretical formalism (M_{1f} depending on M_{1i}) is not far from reality.

4.2. The inverse mass ratio $q^{-1} = M_1/M_2$

Based on the fact that no specific q relation can be found in the mass bins of set I (cf. Section 3), ninety-four systems of set I are transformed to construct the q^{-1} distribution of set II′. The histogram for II′ is given in Figure 8 for the three cases studied in this paper (C, NC50, NC100). The distribution of set II is given for comparison. The most important features of the comparison are summarized in Table IV: the bin where the absolute maximum occurs, the fraction of systems in the bins where most of the observed systems occur, the fraction of systems in the bins where no observed systems occur. The best quantitative correspondence with set II is found with case NC100. The only problem is the appearance of eleven systems with mass ratio $q^{-1} > 0.6$. These systems form two groups: eight massive systems ($M_{1i} \simeq$ 7.2 $M_\odot$) with initial mass ratio $\leqslant 0.5$ and a few small systems ($M_{1i} \sim 1\ M_\odot$) with mass ratios in the range 0.5–0.8. The discrepancy with the observations (to a smaller extent, also present in the two other cases) may indicate that external mass loss is not the same for all systems – i.e., the rate is influenced by the parameters M_1, q and P.

As mentioned in Section 3, the mass ratio distribution of set I shows one major selection effect, namely the underabundance of low mass ratio systems. In order to find the influence of this effect upon the final distribution and on the comparison, we transformed Trimble's (1974) initial distribution as follows.

Each bin of the distribution is transformed into a new bin through the relation

$$q_f^{-1} = (q_i k + \beta k - \beta)^{-1} \tag{6}$$

derived from Equation (3), with $k = M_{1i}/M_{1f}$. The value of k varies with M_{1i} and with the considered case of mass transfer (Equations (1) and (2)), but calculations for M_{1i} ranging from 1 $M_\odot$ to 15 $M_\odot$ show that the following values of k may be used as

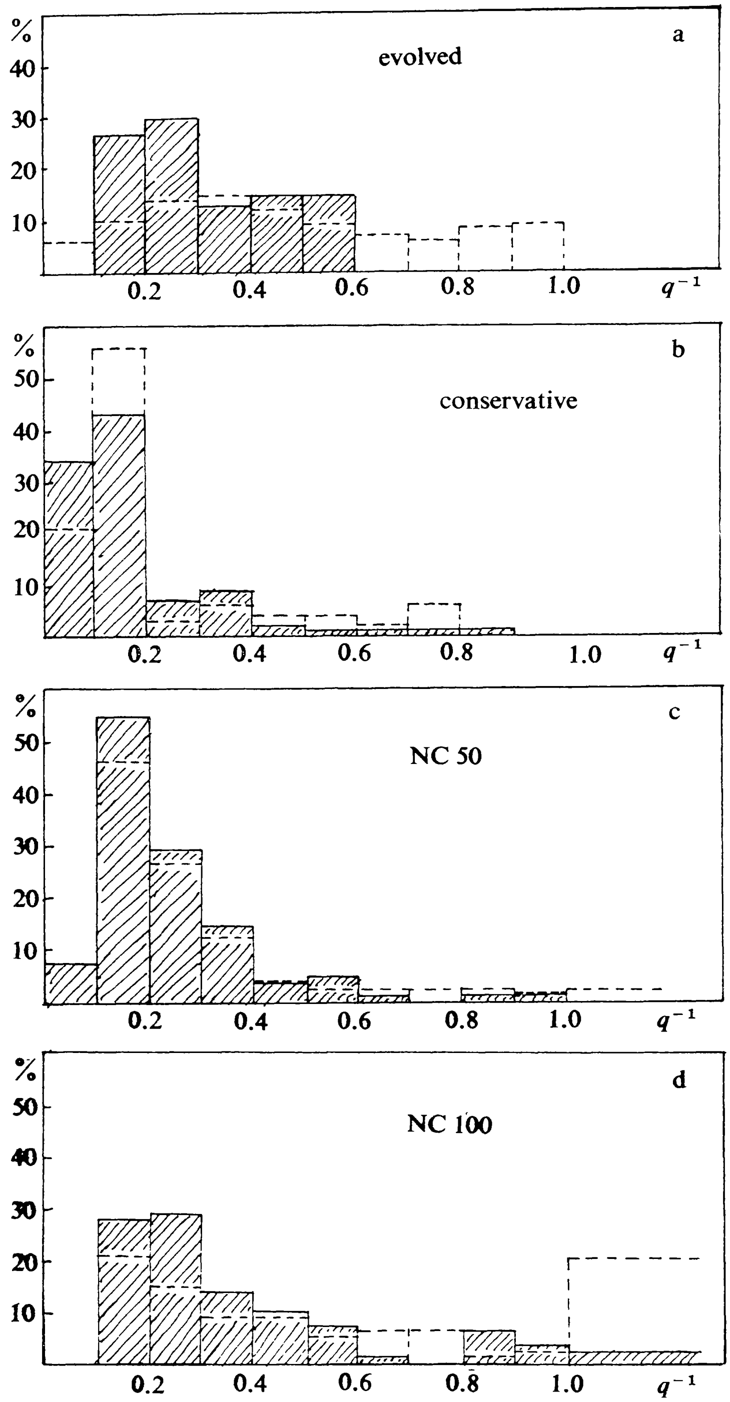

Fig. 8. Distributions of $q^{-1} = M_1/M_2$ for close binary systems after mass exchange. (a) Observed evolved systems; (b) transformed systems (conservative mode); (c) transformed systems (mode NC50); (d) transformed systems (mode NC100). The full lines represent the distributions of the systems in this paper, the dashed lines represent the results of Trimble (1974).

TABLE IV

Comparison of characteristics of the q^{-1} distributions of set II′, with the distribution of observed evolved systems (set II)

Group	Maximum $q_1 < q^{-1} \leqslant q_2$	% of max.	% in interval $0.1 < q^{-1} \leqslant 0.3$	% systems with $q^{-1} > 0.6$
Observations	0.2–0.3	30	57	0
Conservative	0.1–0.2	43	50	3
NC50	0.1–0.2	55	74	3
NC100	0.2–0.3	29	57	11

a representative average:

$$k_{\mathrm{A}} = 2.3 \quad (\text{with } k_{\mathrm{A,min}} = 2.2 \quad \text{and} \quad k_{\mathrm{A,max}} = 2.9)$$

for mass transfer following case A of mass exchange, and

$$k_{\mathrm{B}} = 6.2 \quad (\text{with } k_{\mathrm{B,min}} = 3.2 \quad \text{and} \quad k_{\mathrm{B,max}} = 8.3)$$

for mass transfer following case B of mass exchange.

The chosen values of k correspond to an initial mass of 2 $M_\odot$. Using these two values we transformed the unevolved set of Trimble's distribution by the C, NC50 and NC100 modes. The transformed bins have boundaries different from the original division. The new distribution with bins equal to the original division was formed using proportional fractions of the transformed bins and assuming a composition of 20% case A systems and 80% case B systems (a mixture of 30% case A and 70% case B would lead to quite similar results). The results are given in Figure 8, together with the observed evolved systems both of Trimble's paper and of the present study. As can be seen, the overall agreement with the corresponding distributions of the present study is remarkable. The reason is that the bin |0.2, 0.3| of Trimble's distribution (with the peak value) is transformed into $q^{-1} \sim 0.15$, ~ 0.24 and |0.8, 1.6| for case B (resp. mode C, NC50 and NC100) and into ~ 0.53, ~ 0.82 and |1.4, 2.2| for case A (resp. mode C, NC50 and NC100); i.e., low inverse mass ratios are favoured (except for NC100), taking into account that the majority of systems are transformed following case B. This tendency is found for a large range of initial mass ratios.

One remark must be made about the observed evolved systems (Figure 8). The large number of evolved systems in Trimble's distribution may well be overestimated due to her definition of this group. On the basis of simple evolutionary arguments one expects about 10% of all binaries in the evolved stage. In Trimble's distribution the ratio of the number of evolved to unevolved systems is 1.2. In this context, features of this distribution must be regarded carefully with respect to the contributing systems. An extensive investigation of this problem is in progress.

4.3. The period

Using the same arguments as used in the beginning of Section 4.2, ninety-four systems are transformed to construct the period histogram. This is shown in Figures 9a to 9f, with a summarized analysis given in Table V. The table gives the bin where the maximum occurs and the value of that maximum, the fraction of systems in the range where the bulk of observed evolved systems is found, and the fraction in the period range where almost no observed systems are found. As can be seen, the conservative case disagrees completely with observations while the modes with $\alpha = 1$ show a very small number of systems in the region where the maximum of observed systems occurs.

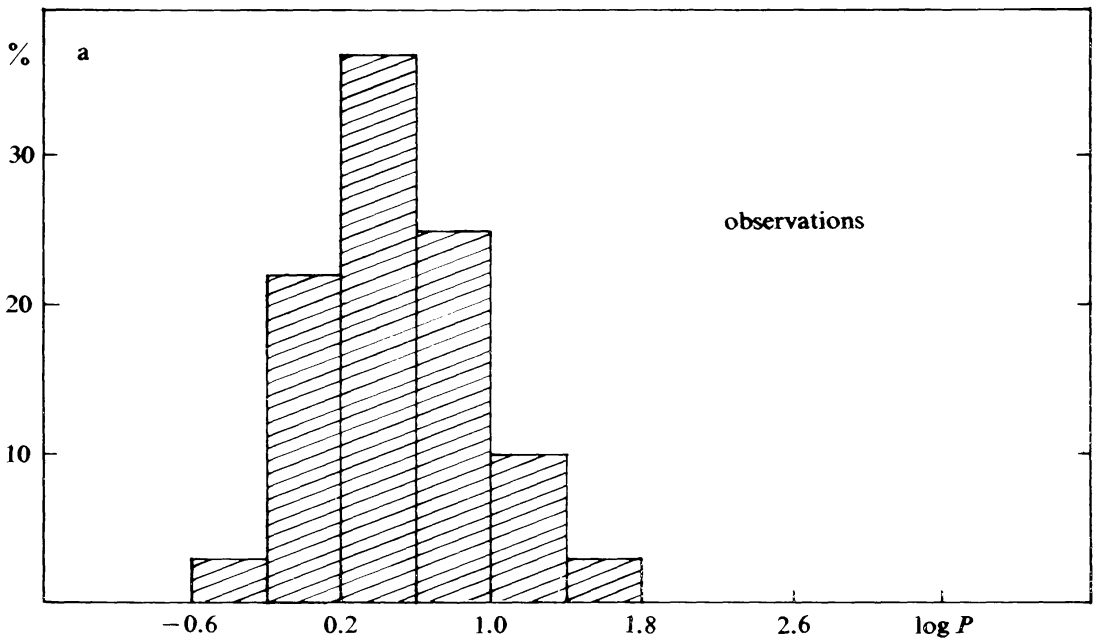

Fig. 9a. Distribution of log P for close binary systems after mass exchange: observations (set II).

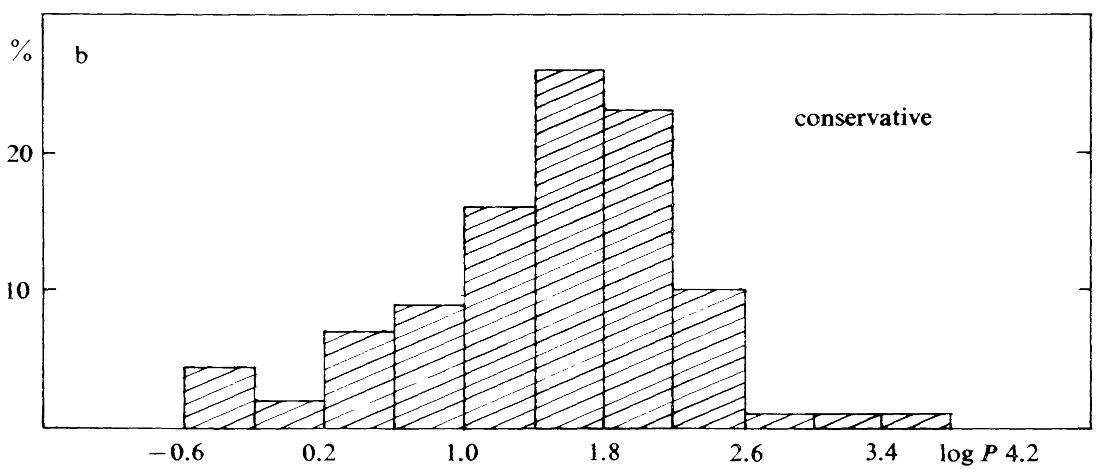

Fig. 9b. Distribution of log P for close binary systems after mass exchange: conservative mode ($\beta = 1$)

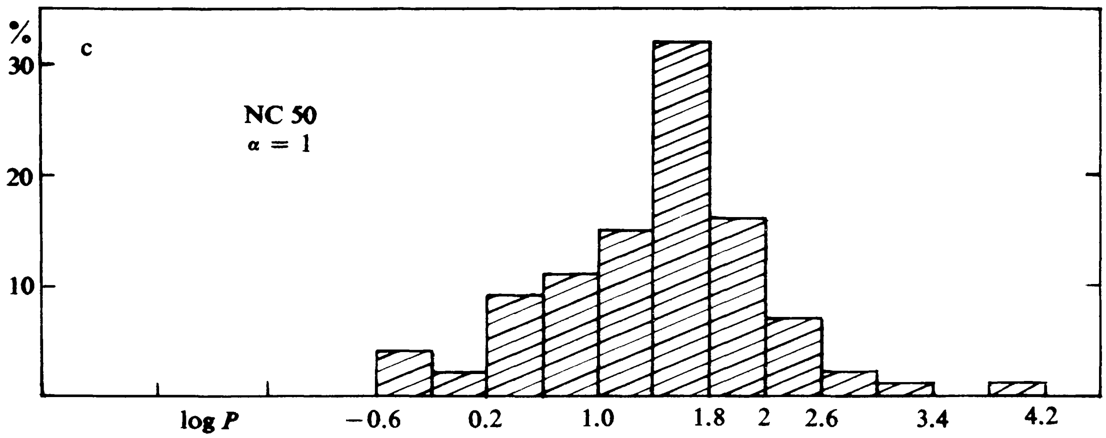

Fig. 9c. Distribution of log P for close binary systems after mass exchange: mode NC51 ($\beta = 0.5$, $\alpha = 1$).

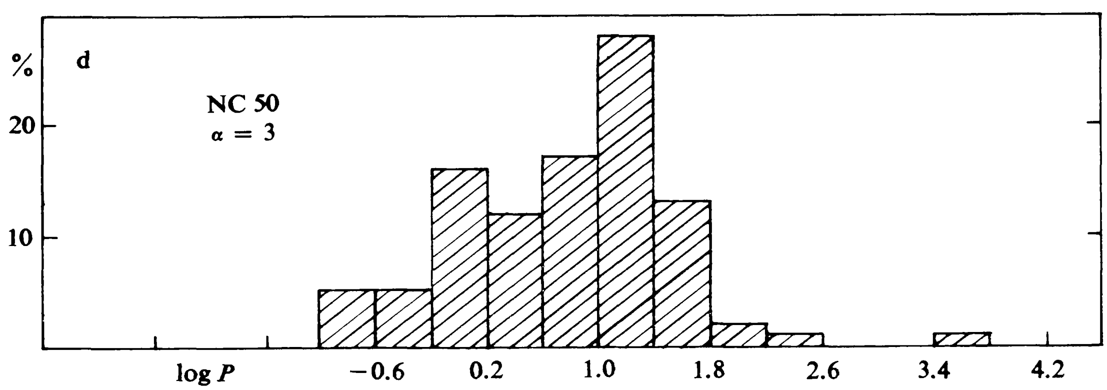

Fig. 9d. Distribution of log P for close binary systems after mass exchange: mode NC53 ($\beta = 0.5$, $\alpha = 3$).

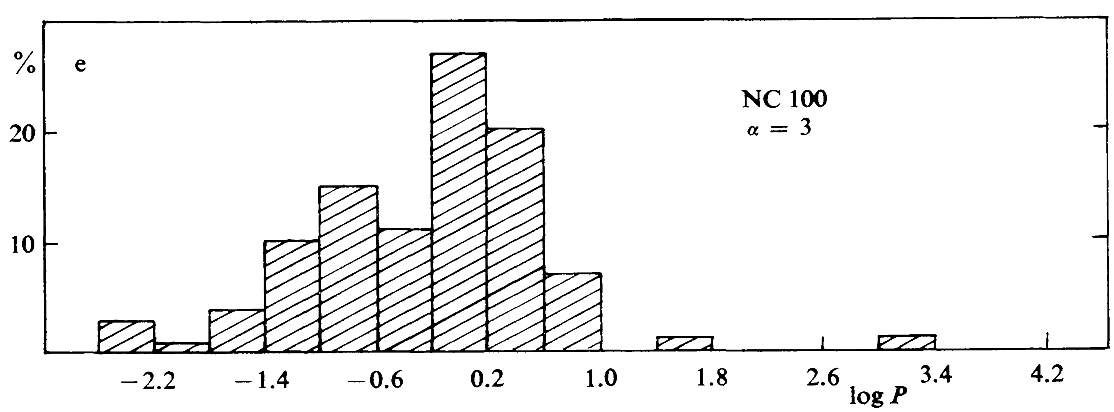

Fig. 9e. Distribution of log P for close binary systems after mass exchange: mode NC101 ($\beta = 0.0$, $\alpha = 3$).

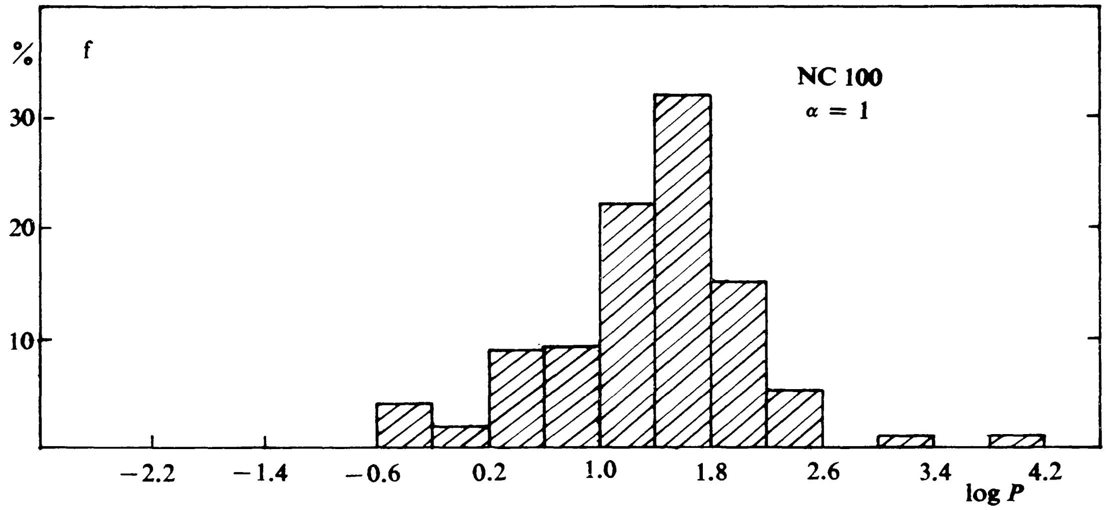

Fig. 9f. Distribution of $\log P$ for close binary systems after mass exchange: mode NC103 ($\beta = 0.0$, $\alpha = 1$).

Moreover, in these cases too many systems appear with large periods (25 d $\leqslant P \leqslant$ 60 d) compared to observations. The first discrepancy is removed with the $\alpha = 3$ mode, while the second only disappears when all the transferred mass is removed from the system. In this case, NC103, the maximum is still displaced by one $\log P$ bin ($\Delta \log P = 0.2$) with regard to the observations. This can be resolved by assuming that not all, but say some 80% of the transferred mass is leaving the system. Such an assumption would also strongly decrease the excess of transformed short period systems (for NC103, 44% with $\log P < -0.2$).

The validity of the results of the comparison of the period distribution is based upon the following arguments. The period distributions of set I and set II show similar characteristics, a maximum in the range $0.2 < \log P \leqslant 0.6$, an underabundance of systems with $\log P > 1.0$, and an absence of systems with $\log P$ smaller than -0.6 (together with very few systems in the range $-0.2 < \log P \leqslant 0.2$). For periods $P <$

TABLE V

The same as Table IV, but for the period distribution

Group	Maximum $\log P_1 < \log P \leqslant \log P_2$	% of max.	% in interval $-0.2 < \log P \leqslant 1.0$	% systems with $\log P > 1.4$
Observations	0.2–0.6	37	84	3
Conservative	1.4–1.8	26	18	62
NC51	1.4–1.8	32	22	59
NC53	1.0–1.4	28	45	17
NC101	1.4–1.8	32	20	54
NC103	−0.2–0.2	27	54	2

TABLE VI

The ratio P_i/P_f for evolved systems with periods in the range $0.25 < P$ (days) $\leqslant 3.98$. The value β_{max} indicates the maximum value of β to obtain a non-negative solution for M_{2i}; NC81 ($\beta = 0.2$, $\alpha = 1$) and NC83 ($\beta = 0.2$, $\alpha = 3$).

System	β_{max}	C	NC51	NC53	NC81	NC83	NC101	NC103
3.19 + 1.53	0.49	—	—	—	0.62	48.1	0.22	35.9
1.72 + 1.00	0.35	—	—	—	2.26	496.7	0.30	151.1
1.81 + 0.50	0.55	—	32.72	839.9	0.19	18.5	0.08	16.5
2.03 + 0.39	0.73	—	0.37	5.6	0.07	3.3	0.04	3.9
1.68 + 0.40	0.59	—	3.95	89.2	0.12	10.3	0.06	10.3
3.13 + 0.90	0.67	—	1.69	26.7	0.17	8.8	0.09	9.5
2.08 + 0.87	0.45	—	—	—	0.60	78.3	0.17	49.3
2.18 + 1.03	0.43	—	—	—	0.81	105.1	0.21	60.6
2.96 + 1.31	0.52	—	—	—	0.52	41.6	0.19	32.2
8.01 + 2.82	0.89	—	0.64	5.2	0.22	4.7	0.15	5.7
1.88 + 0.49	0.57	—	8.1	189.7	0.15	13.6	0.07	12.9
2.22 + 1.18	0.41	—	—	—	1.13	154.3	0.26	77.5
2.96 + 0.36	1	1.18	0.04	0.3	0.02	0.4	0.02	0.6
4.65 + 1.43	0.77	—	0.82	9.3	0.18	6.0	0.11	7.0
1.94 + 0.45	0.63	—	1.75	34.6	0.11	7.8	0.06	8.2
1.82 + 0.44	0.60	—	3.45	75.1	0.13	10.1	0.06	10.1
2.33 + 0.50	0.70	—	0.59	9.4	0.09	4.6	0.05	5.2
3.15 + 0.74	0.74	—	0.53	7.2	0.10	4.4	0.06	5.2
2.56 + 0.35	0.90	—	0.06	0.5	0.03	0.7	0.02	1.0
4.27 + 1.60	0.66	—	3.15	43.9	0.29	12.8	0.15	13.1

1.6 days, our distribution may be assumed to be close to reality, so the last characteristic is independent of selection effects. The shifts of the maximum peaks in the period distribution of set I to the maximum peaks in the transformed set are from 2.5–40 days (C, NC51, NC101), 2.5–16 days (NC53), or 2.5–0 days (NC103).

Using the inverse of Equations (1) and (2) the observed evolved systems with periods in the range $-0.6 < \log P \leqslant 0.4$ can be transformed into unevolved systems, and the ratio of initial to final period P_i/P_f can be calculated for the different modes. First, the maximum value of β is computed for which a non-negative value for the initial mass of the secondary is obtained. If $\beta < 1$, then conservative mass exchange is not possible. In Section 4.2 we suggested that a mass loss of 80% is not unlikely, so P_i/P_f is also computed for $\beta = 0.2$ (modes NC81 and NC83). A few systems with $P < 3.98$ days were omitted as they have primary mass smaller than 0.3 $M_\odot$ (i.e., smaller than the limiting final mass in the transformation equations). The results for the twenty remaining systems are shown in Table VI.

Only one system has a solution in the conservative case. The others clearly have evolved with external mass loss. The average value of β_{max} is 0.6. From the results it is also clear that for values of $\beta \neq 1$, the values $\alpha = 1$ and $\alpha = 3$ lead, respectively, to a period decrease (if one goes from P_f to P_i) and a period increase (with few exceptions for both values). For simplicity we restrict ourselves to the mode NC81 (also because

$\beta = 0.5$ does not hold for all systems in the sample). For $\alpha = 1$ the initial periods are, on average, 0.4 times smaller than the present periods. For the majority of evolved systems with $\log P \leqslant 0.4$, this would lead to initial periods of ~ 1 day. As mentioned, this phenomenon is not observed (cf. Figure 4). From completeness of the distribution in the small period range, we may therefore conclude that the external mass loss is coupled with a large amount of angular momentum loss ($\alpha > 1$). For small period systems the angular momentum loss should be such that the final periods should be of the same order of magnitude.

Appendix

List of Observed Close Binary Systems used in the Investigation

Cat. No.		P (d)	M_1	M_2	$\log L_1$	$\log L_2$	$\log T$	$\log T$	References
HD	1337	3.524	23.00	18.00	4.642	5.207	4.460	4.505	(1)
	1486	1.813	3.10	1.39	2.070	1.028	4.029	3.787	(1)
	4161	4.467	2.11	1.27	1.511	0.433	3.947	3.813	(1)
	5679	2.493	3.19	1.53	2.066	0.875	4.079	3.678	(1)
	6882	1.670	6.10	3.00	2.631	1.746	4.164	4.046	(1)
	12 211	0.972	1.72	1.00	1.044	0.000	3.947	3.642	(1)
	16 506	0.685	2.04	0.64	1.353	−0.212	3.958	3.741	(1)
	16 907	1.428	2.90	1.18	1.964	—	4.029	—	(1)
	17 034	6.864	5.27	0.83	2.917	1.668	4.223	3.760	(1)
	17 138	1.195	1.81	0.50	1.139	0.140	3.958	3.672	(1)
	18 541	2.648	2.03	0.39	1.339	0.462	3.947	3.682	(1)
	21 985	2.664	2.02	0.25	1.330	0.398	3.982	3.697	(1)
	25 204	3.953	8.77	2.07	3.656	2.145	4.253	3.962	(1)
	25 487	2.769	3.01	0.82	2.025	—	4.079	—	(1)
	25 833	2.029	5.06	4.47	2.679	2.446	4.193	4.153	(1)
	26 609	0.321	0.97	0.93	−0.178	−0.303	3.753	3.746	(1)
	32 068/9	972.162	8.30	5.60	3.832	2.297	3.613	4.149	(1)
	33 088	1.333	6.73	5.32	3.190	2.431	4.253	4.099	(1)
	33 357	1.210	10.80	5.66	3.361	2.448	4.238	4.058	(1)
	34 029	104.023	2.88	2.88	1.912	1.812	3.740	3.813	(2)
	34 333	4.066	21.47	21.42	—	—	4.274	4.086	(3), (4)
	34 364	4.135	2.48	2.29	1.587	1.454	4.029	3.994	(1)
	35 311	3.431	3.32	2.51	1.872	1.484	4.093	4.127	(4)
	35 652	1.811	17.40	11.80	4.337	4.417	4.447	4.384	(4)
	35 921	4.003	21.53	8.07	4.608	4.248	4.492	4.492	(4)
	36 486	5.732	26.90	10.20	5.544	4.458	4.491	4.390	(1)
	36 695	1.485	6.70	3.35	3.325	2.048	4.354	4.201	(1)
	37 513	4.181	1.17	1.11	0.360	0.296	3.792	3.781	(5)
	39 220	2.933	3.50	1.54	2.253	0.853	3.982	3.886	(1)
	40 183	3.960	2.34	2.25	1.564	1.533	3.958	3.969	(1)
	44 691	9.944	2.08	1.62	1.387	1.067	3.914	3.881	(1)
	44 701	1.190	9.01	5.96	2.898	1.985	4.193	4.042	(1)
	44 982	0.593	1.15	0.76	0.268	−0.540	3.760	3.656	(1)
	46 052	2.525	1.81	1.75	1.173	1.103	3.914	3.897	(1)

(*continued*)

Cat. No.	P (d)	M_1	M_2	log L_1	log L_2	log T	log T	References
48 915	18277.110	2.45	0.74	1.348	−2.872	4.041	4.579	(2)
57 060	4.393	23.00	19.00	5.377	4.777	4.568	4.568	(4)
57 167	1.136	1.65	0.24	0.963	−0.305	3.889	3.615	(1)
58 713	9.301	2.69	0.58	1.835	1.122	3.958	3.659	(1)
61 421	14694.008	1.23	0.39	0.816	−3.272	3.816	3.944	(2)
65 607	5.905	3.46	1.53	1.719	1.866	3.735	3.914	(1)
65 818	1.455	19.10	11.30	4.001	3.236	4.354	4.188	(1)
72 257	2.904	1.23	1.12	0.475	0.239	3.833	3.813	(1)
76 943	8108.000	0.78	0.34	0.536	−0.292	3.816	3.978	(2)
77 464	6.892	5.62	5.42	3.217	3.177	4.344	4.344	(4)
78 014	2.282	1.68	0.40	1.001	−0.070	3.901	3.546	(1)
83 950	0.334	1.29	0.86	0.251	−0.197	3.793	3.803	(1)
91 636	2.445	2.74	1.05	1.862	0.775	3.958	3.797	(1)
92 109	0.600	0.99	0.92	0.143	0.090	3.777	3.765	(1)
93 033	3.063	3.13	0.90	1.997	1.088	4.079	3.746	(1)
96 314	2.268	3.71	2.70	2.316	1.740	4.127	4.127	(4)
98 230/I	3.981	1.48	1.15	−0.056	−0.240	3.744	3.744	(2)
100 213	1.387	23.50	15.80	4.870	4.480	4.530	4.470	(6)
104 350	0.643	2.16	0.67	1.451	0.203	3.958	3.823	(1)
106 400	0.408	1.38	0.58	0.098	−0.517	3.719	3.716	(1)
114 519	4.798	1.42	1.35	0.672	0.689	3.611	3.839	(1)
116 658	4.014	10.90	6.80	2.788	2.192	4.342	4.267	(7)
121 648	4.992	1.76	1.67	1.076	1.032	3.841	3.841	(3), (4)
121 909	0.817	0.87	0.86	0.007	0.150	3.756	3.793	(1)
128 220	870.000	2.50	2.70	—	—	4.504	3.772	(8)
128620/I	29190.780	1.45	0.58	0.032	0.516	3.762	3.700	(2)
132 742	2.327	2.96	1.31	1.976	0.770	3.982	3.675	(1)
133 640	0.268	0.96	0.49	−0.146	−0.394	3.763	3.783	(1)
136 175	3.452	6.76	2.57	2.276	1.824	4.215	4.033	(2)
139 319	2.807	2.08	0.87	1.437	0.788	3.922	3.674	(1)
130 265	1.701	2.68	1.19	1.824	0.541	3.947	3.681	(1)
150 484	0.453	1.53	0.61	0.398	0.070	3.774	3.785	(1)
150 680	12557.295	1.07	0.76	0.672	−0.380	3.898	3.974	(2)
150 708	4.630	4.03	2.38	0.948	0.811	3.763	3.608	(1)
151 890	1.446	14.00	9.24	3.689	2.680	4.333	4.058	(1)
153 345	1.199	2.18	1.03	1.467	0.650	3.982	3.756	(1)
153 751	39.481	2.36	1.19	1.865	0.354	3.700	3.745	(1)
154 676	4.184	1.13	1.11	0.357	0.381	3.793	3.793	(1)
155 937	0.422	1.43	0.43	0.699	−0.354	3.860	3.797	(1)
156 247	1.677	5.02	4.52	2.766	2.613	4.193	4.169	(1)
156 633	2.051	8.01	2.82	3.532	2.355	4.312	4.025	(1)
156 965	2.060	2.06	1.77	1.196	0.797	3.929	3.879	(1)
161 783	3.170	6.80	5.80	2.664	2.476	4.280	4.240	(9)
163 175	1.549	1.88	0.49	1.202	−0.449	3.947	3.527	(1)
163 181	12.004	14.26	24.83	4.216	3.376	4.433	4.477	(4)
163 611	0.410	1.30	0.44	0.369	−0.181	3.833	3.803	(1)
163 930	3.993	1.22	1.10	0.674	0.460	3.833	3.673	(1)
165 341	32036.078	1.00	0.72	−0.316	−1.028	3.949	3.936	(2)
167 647	2.416	5.93	2.16	3.096	1.922	4.193	3.948	(1)

(continued)

Cat. No.	P (d)	M_1	M_2	log L_1	log L_2	log T	log T	References
170 470	2.197	2.38	1.98	1.656	1.359	3.982	3.957	(1)
170 757	1.779	2.72	2.32	1.718	1.455	4.000	3.979	(1)
173 787	8.896	12.10	4.70	3.372	3.057	4.253	3.975	(1)
175 227	10.550	3.72	4.17	2.356	2.300	4.215	4.246	(2)
176 853	1.849	11.75	7.41	2.916	2.400	4.215	4.127	(2)
177 708	2.408	1.85	1.50	1.447	1.029	3.958	3.888	(1)
178 125	1.302	4.54	0.64	3.688	1.105	4.079	3.909	(1)
179 890	2.178	1.02	0.93	0.060	−0.147	3.777	3.693	(1)
180 939	4.477	4.65	1.45	2.961	1.013	4.193	3.638	(1)
181 987	2.455	5.40	2.27	3.180	2.129	4.223	3.965	(1)
185 507	1.950	6.80	5.40	3.058	2.716	4.253	4.167	(1)
187 399	27.970	18.00	13.00	4.497	4.017	3.593	4.433	(4)
187 879	12.426	13.90	7.95	4.898	3.588	4.333	4.287	(1)
187 949	1.183	2.22	1.18	1.501	0.414	3.958	3.693	(1)
189 371	1.805	2.74	0.90	1.864	0.778	3.982	3.773	(1)
190 786	2.347	2.37	1.60	1.004	0.162	3.947	3.801	(1)
190 967	6.520	17.38	22.39	3.916	3.836	3.967	3.816	(2)
192 577/8	3784.000	9.21	6.20	3.837	3.480	3.623	4.223	(1)
192 909/10	1140.800	20.89	7.41	2.484	1.404	3.644	4.127	(2)
193 576	4.212	25.60	10.10	5.278	—	4.602	4.568	(1)
193 611	2.900	14.80	14.60	3.436	3.316	4.477	4.477	(7)
196 628	0.718	1.39	1.17	1.576	0.930	4.029	3.845	(1)
197 433	0.278	0.79	0.26	−0.146	−1.171	3.753	3.669	(1)
198 287/8	13.597	9.65	8.00	3.791	3.090	3.958	3.860	(1)
198 846	2.996	17.80	17.50	4.426	4.420	4.477	4.477	(1)
199 454	3.436	2.96	0.36	2.014	0.650	4.029	3.686	(1)
200 391	0.698	1.23	1.13	0.302	0.142	3.777	3.748	(1)
205 234	8.446	1.80	1.79	1.346	1.179	3.860	3.875	(1)
206 155	2.628	1.94	1.20	1.350	0.317	3.936	3.793	(1)
206 821	4.428	1.17	1.07	1.099	0.117	3.982	3.765	(1)
207 956	10.623	2.01	0.32	2.001	0.885	3.929	3.674	(1)
208 816	7450.000	84.40	41.30	5.696	4.610	3.544	3.963	(1)
209 147	1.605	2.01	1.50	1.149	0.661	3.958	3.862	(1)
210 334	1.983	1.30	1.29	0.606	0.478	3.687	3.763	(1)
215 661	2.142	4.10	1.90	2.478	0.832	4.134	3.769	(1)
216 014	1.775	16.10	13.90	3.956	3.613	4.354	4.268	(1)
216 598	0.321	1.05	0.89	−0.180	−0.289	3.739	3.760	(1)
218 066	2.729	9.98	9.77	3.498	3.078	4.253	4.210	(1)
219 815	3.220	2.48	1.32	1.692	1.243	3.914	3.801	(1)
221 253	6.066	6.95	2.06	3.454	1.284	4.253	3.942	(1)
222 217	2.337	1.72	0.27	1.044	0.511	3.901	3.745	(1)
224 930	9595.118	0.78	0.85	2.156	2.008	3.756	4.146	(2)
227 696	3.879	17.80	13.78	3.676	3.468	4.391	4.391	(7)
228 854	1.885	37.40	32.80	4.935	4.320	4.505	4.380	(1)
228 911	1.874	13.70	12.10	4.080	2.729	4.312	4.066	(1)
250 371	2.866	4.65	1.43	2.691	1.498	4.193	3.821	(1)
276 247	13.199	2.83	0.46	1.917	1.328	3.929	3.702	(1)
285 892	0.416	1.71	0.92	0.717	−0.070	3.876	3.830	(1)
352 682	4.806	2.36	0.47	1.609	0.785	3.982	3.640	(1)

(*continued*)

Cat. No.	P (d)	M_1	M_2	log L_1	log L_2	log T	log T	References
ADS 9975	39808.598	0.86	0.65	—	—	3.662	3.591	(10)
BD −21°311	0.317	1.29	0.67	−0.230	−0.626	3.753	3.710	(1)
− 8°1050	0.423	0.47	0.28	−0.203	−0.324	3.760	3.763	(1)
− 2°2331	5.049	2.30	1.00	3.011	1.422	4.193	3.787	(1)
15°4915	0.375	1.33	1.07	0.493	0.154	3.845	3.835	(1)
17°3117	0.912	2.44	0.90	1.665	—	3.958	—	(1)
24°2475	0.339	1.64	0.78	−0.026	−0.550	3.719	3.723	(1)
26°1981	1.686	1.91	0.63	1.276	—	3.947	—	(1)
32°1582	0.814	0.56	0.56	−1.278	−1.278	3.556	3.556	(1)
32°4756	4.123	1.81	0.39	1.137	0.564	3.876	3.641	(1)
36°5017	0.332	1.82	1.13	0.127	−0.048	3.753	3.755	(1)
46°740	0.849	1.32	0.32	0.540	−0.166	3.860	3.683	(1)
47°3639	1.677	3.06	2.60	3.118	1.878	4.253	4.012	(1)
52°	0.629	1.52	1.00	0.455	−0.304	3.793	3.685	(1)
56°1395	1.687	1.94	0.45	1.265	−0.022	3.982	3.704	(1)
73°533	1.357	1.82	0.44	1.144	−0.114	3.929	3.611	(1)
76°286	3.306	2.33	0.50	1.587	0.300	3.914	3.597	(1)
TX Cnc	0.383	1.29	0.65	−0.136	−0.136	3.793	3.793	(1)
V Sge	0.514	2.91	0.77	—	—	—	—	(1)
RW CrB	0.726	1.60	0.42	0.912	—	3.876	—	(1)
V 1073 Cyg	0.786	1.37	0.47	1.361	0.544	3.947	3.847	(1)
GK Cep	0.936	2.72	2.50	1.567	1.465	3.958	3.945	(1)
AU Pup	1.126	2.77	2.19	1.868	1.496	3.982	3.940	(1)
IM Aur	1.247	2.97	0.89	1.999	0.203	4.029	3.679	(1)
CM Dra	1.268	0.24	0.21	−3.192	−3.252	3.509	3.509	(4)
HS Hya	1.568	1.34	1.29	0.496	0.412	3.824	3.824	(4)
RS Cha	1.670	2.12	1.76	1.426	1.122	3.929	3.904	(1)
V 526 Sgr	1.919	2.11	1.66	1.432	1.019	3.982	3.917	(1)
GL Car	2.422	5.89	5.77	3.091	3.055	4.253	4.244	(1)
YY Sgr	2.628	2.36	2.29	1.579	1.531	3.982	3.986	(1)
XY Cet	2.781	2.27	2.07	1.528	1.375	3.982	3.951	(1)
RZ Cha	2.832	1.51	1.51	0.937	0.937	3.818	3.818	(6)
β Per	2.867	3.15	0.74	2.082	0.797	4.079	3.696	(1)
TL Mi	3.020	2.56	0.35	1.746	−0.159	3.982	3.462	(1)
U Sge	3.381	4.27	1.60	2.593	0.573	4.134	3.567	(1)
CD Tau	3.435	1.41	1.30	0.676	0.620	3.810	3.810	(4)
RU Mon	3.585	2.34	0.63	1.595	1.491	4.029	4.010	(1)
V 624 Her	3.895	2.06	1.85	1.604	1.284	3.967	3.913	(4)
LY Aur	4.003	21.60	8.10	5.270	4.960	4.530	4.520	(4)
MY Cyg	4.005	17.70	17.42	3.304	3.304	3.913	3.936	(4)
SZ Cen	4.108	2.28	2.32	1.710	1.840	3.910	3.890	(6)
UW CMa	4.393	47.40	37.20	5.601	4.561	4.544	4.307	(1)
BM Ori	6.471	5.27	2.87	2.918	2.262	4.312	3.877	(1)
V 1143 Cyg	7.641	1.46	1.44	0.598	0.513	3.833	3.842	(1)
α CrB	17.360	2.75	0.94	1.832	−0.095	3.982	3.761	(1)
AR Mon	21.208	2.68	0.80	1.436	1.396	3.679	3.740	(4)

(continued)

Cat. No.	P (d)	M_1	M_2	log L_1	log L_2	log T	log T	References
RZ Cno	21.643	3.19	0.54	1.248	1.132	3.670	3.653	(4)
KU Cyg	38.439	4.69	0.81	1.705	1.721	3.876	3.583	(1)
μ^1 Sco	1.440	13.96	9.23	3.793	3.041	4.364	4.188	(4)

References:
(1) Giannone, P. and Giannuzzi, M. A.: 1974, *Astrophys. Space Sci.* **26**, 289.
(2) Batten, A. H. and Ovenden, M. W.: 1968, *Monthly Notices Roy. Astron. Soc.* **140**, 81.
(4) Lacy, C. H.: 1979, *Astrophys. J.* **228**, 817.
(5) Clausen, J. V. and Gronbech, B.: 1976, *Astron. Astrophys.* **58**, 131.
(6) Andersen, J.: 1975, *Astron. Astrophys.* **45**, 107.
(7) Levato, H.: 1974, *Astron. Astrophys.* **35**, 259.
(8) Joy, J.: 1977, *Astron. Astrophys.* **57**, 449.
(9) Kuiper, G. F. G.: 1971, *Astron. Astrophys.* **14**, 70.
(10) Dommanget, J. and van Dessel, E. L.: 1970, *Astron. Astrophys.* **6**, 423.

References

Abt, H. A. and Levy, S. G.: 1976, *Astrophys. J. Suppl.* **30**, 273.
Abt, H. A. and Levy, S. G.: 1978, *Astrophys. J. Suppl.* **36**, 241.
Aitken, R. G.: 1964, in *The Binary Stars*, Dover Publ., New York, ch. 5.
Batten, A. H.: 1970, *Publ. Astron. Soc. Pacific* **82**, 574.
de Grève, J. P.: 1976, Ph. D. Thesis, V.U.B., Brussels.
de Grève, J. P., de Loore, C. and van Dessel, E. L., 1978, *Astrophys. Space Sci.* **53**, 105.
Flower, J.: 1977, *Astron. Astrophys.* **54**, 31.
Horn, J., Kříž, S. and Plavec, M.: 1969, *Bull. Astron. Inst. Czech.* **20**, 193.
Iben, I.: 1967, *Astrophys. J.* **147**, 650.
Lang, R.: 1974, *Astrophysical Formulae*, Springer Verlag, Berlin, p. 563.
Lequeux, J.: 1979, *Astron. Astrophys.* **71**, 1.
Paczynski, B.: 1966, *Acta Astron.* **16**, 231.
Paczynski, B.: 1971, *Ann. Rev. Astron. Astrophys.* **9**, 183.
Pédoussant, A. and Ginestet, N.: 1971, *Astron. Astrophys. Suppl.* **4**, 253.
Pédoussant, A. and Carquillat, J. M.: 1973, *Astron. Astrophys. Suppl.* **10**, 105.
Pédoussant, A. and Nadal, R.: 1977, *Astron. Astrophys. Suppl.* **27**, 55.
Plavec, M.: 1968, *Astrophys. Space Sci.* **1**, 239.
Plavec, M.: 1973, in A. H. Batten (ed.), *Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems*, D. Reidel Publishing Co., Dordrecht, p. 216.
Thomas, H. C.: 1977, *Ann. Rev. Astron. Astrophys.* **15**, 127.
Trimble, V.: 1974, *Astron. J.* **79**, 967.
Trimble, V.: 1978, *Observatory* **98**, 163.
van den Heuvel, E. P. J.: 1976 in P. Eggleton, S. Mitton and J. Whelan (eds.), *Structure and Evolution of Close Binary Systems*, D. Reidel Publishing Co., Dordrecht, p. 303.
Vanbeveren, D., de Grève, J. P., van Dessel, E. L. and de Loore, C.: 1979, *Astron. Astrophys.* **73**, 19.
Vanbeveren, D. and de Grève, J. P.: 1979, *Astron. Astrophys.* **77**, 295.
Vanbeveren, D. and de Loore, C.: 1979, *Astron. Astrophys.* (in press).
Webbink, R. F.: 1975, Ph. D. Thesis, Cambridge, England.
Ziolkowski, J.: 1970, *Acta Astron.* **20**, 213.