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CHANGES IN THE ORBITAL PERIODS OF CLOSE BINARY STARS

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ABSTRACT

A number of close binary stars show erratic changes in their orbital periods on time scales of order 5–10 yr. Recently two papers have each proposed that the period changes are the result of changes in the quadrupole moment of one star, caused in turn by an alteration of the internal structure of that star. Magnetic pressure, which either distorts the shape of the star or changes its tidally induced quadrupole moment, is suggested as the driving force behind the alteration. However, neither paper discusses the feasibility of such distortions which require an input of energy. We estimate the amount of energy required to distort one component of a binary and match the observed period changes. The rate at which energy is produced or lost is governed by the thermal time scale of the star, and our estimates indicate that the observed period changes would take at least 1000 yr for the tidal quadrupole mechanism, and of order 60 yr to match a period change in V471 Tau which took only 4 yr.

Subject headings: stars: binaries — stars: individual (V471 Tauri) — stars: interiors

I. INTRODUCTION

In a pair of recent papers Applegate and Patterson (1987) and Warner (1988) have considered the effect of structural change in one star of a binary system (due, for example, to magnetic cycles) on the orbital period. If one star has a non-negligible quadrupole moment, a change in its internal structure can change the quadrupole moment and hence also the orbital period.

Warner considers the quasi-periodic variations seen in the orbital periods of some cataclysmic variable stars. He proposes that the change in the quadrupole moment required is a result of a change in radius of the tidally distorted secondary star, which is, in turn, the result of a magnetic cycle. Warner relates the tidally induced quadrupole moment to the apsidal motion constant, although he underestimates the change in radius required for a given change in period.

Applegate and Patterson consider V471 Tau which is a 12.5 hr binary system consisting of a K2 V star and a white dwarf. They propose that the change in orbital period of $|\Delta P|/P \approx 2 \times 10^{-6}$ which took place on a time scale of 4 yr around 1972 could be explained in terms of a magnetic cycle within the star leading to a change of the shape of the star (for an alternative explanation see Bois, Lanning, and Mochnacki 1988). In contrast to Warner, Applegate and Patterson reject as negligible the effect of a general enhancement of magnetic pressure within the convection zone leading to expansion of the star and so to a change in the tidally induced quadrupole moment. Instead they propose that the magnetic pressure is strongly anisotropic and so gives rise to a quadrupole moment directly.

Neither paper considers whether the required magnetic pressure change is feasible, and in particular whether enough energy can be generated to effect such changes on the observed time scales. The production, and subsequent dissipation and/or expulsion, of magnetic flux around a stellar magnetic cycle requires a net input of energy. The immediate production mechanism for the magnetic flux is presumably some kind of dynamo which derives its energy from fluid dynamical motions within the stellar envelope (e.g., Weiss 1983). Convective motions appear to be the main prerequisite for stellar dynamo action (coupled with differential stellar rotation), and the ultimate energy source for the convection is the nuclear energy source of the star itself. Thus, on average, the rate at which energy is generated in the form of magnetic field cannot exceed the rate at which energy becomes available from the stellar energy source and is, in practice, likely to be much less. As a result, there is a lower limit to the time scale on which the proposed cyclic changes can take place.

Our purpose in this paper is to estimate the energy required for the two mechanisms from simple physics. We find that the magnitude of the observed changes cannot be matched by either mechanism in the time available, suggesting that the problem of period changes in close binaries has yet to be solved. In § II we consider the relation between a change in the radius of one star and the consequent change in the orbital period for a tidally distorted star. In the following section we compute parameters required in this relation for polytropes. In § IV we apply the results of §§ II and III to the systems discussed by Warner and by Applegate and Patterson. In § V we compute the energy and time scale of the anisotropic magnetic pressure mechanism suggested by Applegate and Patterson.

II. THE PERIOD CHANGE DUE TO TIDAL DISTORTION: THE BASIC FORMULA

We consider a binary star system whose stars have masses M_1 and M_2 and are at a separation D in a circular orbit. If the stellar radii, R_1 and R_2 respectively, are small ($R_1 \ll D$, $R_2 \ll D$), then the orbital angular velocity , Ω, is given by

$$\Omega^2 = \Omega_0^2 = \frac{G(M_1 + M_2)}{D^3} . \qquad (1)$$

However, if one of the stars (star 2, for example) is not small, its gravitational potential can no longer be treated as that due to a point mass. The star is no longer spherically symmetric because it is distorted both by its own rotation and also by

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tidal forces from its companion. Thus the actual orbital period, $2\pi/\Omega$, differs slightly from $2\pi/\Omega_0$. Applegate and Patterson (1987) noted that the difference is predominantly due to the quadrupole moment of the star, and Warner (1988) noted that for tidal distortion this difference is related to the apsidal precession period $2\pi/\omega_P$, though his statement that $\Omega - \Omega_0 = \omega_P$ is incorrect. To lowest order in R_2/D, the binary angular frequency is given by

$$\Omega^2 = \Omega_0^2\left\{1 + \left[\frac{1}{3}\left(\frac{\omega}{\Omega_0}\right)^2 + \frac{6}{q}\right]K\left(\frac{R_2}{D}\right)^5\right\}, \quad (2)$$

where ω is the uniform stellar rotation rate, $q = M_2/M_1$ is the mass ratio and K is the usual apsidal constant appropriate to the quadrupole term of star 2 (Cowling 1938; Batten 1973).

We now wish to calculate the change in $P = 2\pi/\Omega$ caused by a slight rearrangement of the internal structure of star 2. The change in star 2 leads to changes in R_2, D, K, and ω. The change in ω is the most problematic. Since the change in star 2 is unlikely to be linear in radius, conservation of angular momentum results in a differential rotation rate, invalidating the assumptions contained in equation (2). Applegate and Patterson (1987) argue that the spin-orbit coupling time scale is long, so that conservation of spin angular momentum is a reasonable assumption, but neither they nor Warner address the problem of the variation of ω. However, since the binary systems we consider have $q \lesssim 1$, and are expected to have $\omega \approx \Omega_0$, it may be a reasonable procedure to neglect the rotational term and its variation in equation (2).

The variation is assumed to conserve individual stellar masses, and the orbital angular momentum (neglecting spin contributions). This yields (cf. Warner's eq. [9])

$$\frac{\Delta P}{P} = -12(\lambda + 5)\frac{K}{q}\left(\frac{R_2}{D}\right)^5\frac{\Delta R_2}{R_2}, \quad (3)$$

where we have assumed

$$\frac{\Delta K}{K} = \lambda\frac{\Delta R_2}{R_2}. \quad (4)$$

We discuss the value of λ below. For tidal distortion, Applegate and Patterson argue that if Q is the quadrupole moment and I the moment of inertia, then $\Delta Q/Q = -\Delta I/I$ (their eq. [9]). If we suppose that $I = kM_2R_2^2$, and that

$$\frac{\Delta k}{k} = \mu\frac{\Delta R_2}{R_2}, \quad (5)$$

then their derivation of ΔP is equivalent to assuming that $(\lambda + 5) = -(\mu + 2)$.

III. ESTIMATION OF λ AND μ

The values of λ and μ can be computed by perturbing a stellar model and then measuring the resulting effect on K, I, and R. There have been several previous studies of the response of a star to similar perturbations (Gilliland 1982; Endal and Twigg 1982). As changes in R are directly observable for the Sun, they have been of central interest in these papers, and turn out to be sensitive to the exact model employed. For example, Gilliland points out that estimates of $d\ln R/d\ln L$ vary by two orders of magnitude and even in sign. Further difficulties lie in the uncertainty of the perturbation to be applied. Since the magnetic field in stars can at best be axially symmetric, a correct treatment of the perturbation requires a two-dimensional stellar model. However, our understanding of any details of the magnetic field and how it may affect the energy transport is so limited that detailed modeling of such complexity would be unjustified. Such uncertainties have led to a variety of different assumptions and a corresponding variety of answers even for one-dimensional models (Gilliland 1982).

Although the radius change is subject to the uncertainties just discussed, it enters as an intermediate step in our computations, relating a change in the quadrupole moment ΔQ with the amount of energy ΔE required to effect it. While a change in radius may only involve a small fraction of the star, a change in the quadrupole moment must involve a significant fraction of the star and require an equivalent amount of energy. Therefore, the uncertainty in ΔR will not translate to uncertainties in the relation between ΔE and ΔQ. For these reasons, we choose here to consider polytropes, which, although a simplification, show the essential behavior of interest.

We consider two simple models: model A is a polytrope with index $n = 1.5$, appropriate to a fully convective star, and model B is a bipolytrope with inner and outer indices $n_1 = 2.5$ and $n_2 = 1.5$, respectively, as a model of a radiative core with an outer convective envelope. We then increase the polytropic constant in an outer mass fraction, f, of each model and measure the resulting changes in K, k, and R_2 for a fixed value of the stellar mass M_2. For model B we made the additional simplification that f also equaled the mass fraction of the star with polytropic index n_2. The change is intended to mimic the additional pressure due to variation of magnetic field strength throughout the convective zone during a magnetic cycle. In Figure 1 we show a plot of density versus radius before and after the change for one particular case. Note that since the pressure must remain continuous throughout the star, the density must be discontinuous as we change the polytropic constant for an outer mass fraction. We plot the values of λ and μ as functions of f in Figure 2. The behavior shown there can be understood as follows: as $f \to 0$, we approach the point where the radius change involves none of the mass. In this circumstance we would expect no change in the moment of inertia I, nor in the orbital period, which implies $\lambda \to -5$ and $\mu \to -2$. As $f \to 1$, the whole star becomes involved and the change is a homologous one implying $\lambda \to 0$ and $\mu \to 0$. The negative values of λ contrast with Warner's argument, based on K scaling as R^5, that $\lambda = 5$. Warner's argument is incorrect

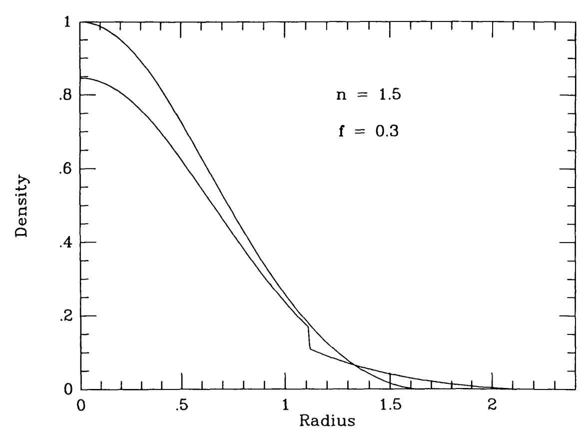

FIG. 1.—The run of density with radius is plotted for a polytropic model with index $n = 1.5$ before and after the polytropic constant was doubled in the outer $f = 0.3$ of the mass.

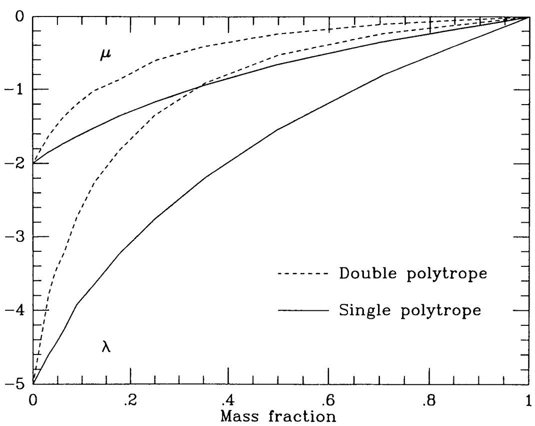

FIG. 2.—The values of the parameters $\lambda = (\Delta K/K)/(\Delta R/R)$ (*lower two curves*) and $\mu = (\Delta k/k)/(\Delta R/R)$, where K is the apsidal constant and k is the dimensionless moment of inertia, are plotted against the mass fraction involved in the radius change. The single polytrope has $n = 1.5$. The double polytrope has outer mass fraction f with $n = 1.5$ and inner mass fraction $1 - f$ with $n = 2.5$.

because K reflects the concentration of mass after scaling the star to a constant radius and is therefore dimensionless.

IV. APPLICATIONS

If the outer mass fraction, f, of the star is raised so that the radius changes by ΔR_2 then we may estimate the energy required to effect the change as

$$\Delta E \approx f E_B \frac{\Delta R_2}{R_2}, \quad (6)$$

where E_B is the stellar binding energy (an exact computation for model B gave a value of ΔE about twice that of eq. [6]). The rate at which energy becomes available to effect such a change is given at most by the stellar luminosity, L. Thus the time scale, t_{CH}, on which change can be effected is given by

$$t_{CH} > \frac{\Delta E}{L} = f \frac{\Delta R_2}{R_2} t_{KH}, \quad (7)$$

where t_{KH} is the Kelvin–Helmholtz time scale for the star as a whole. We use (Lang 1980)

$$t_{KH} = 2 \times 10^7 \left(\frac{M}{M_\odot}\right)^2 \left(\frac{L}{L_\odot}\right)^{-1} \left(\frac{R}{R_\odot}\right)^{-1} \text{ yr}. \quad (8)$$

a) U Geminorum

Warner (1988) finds a cyclic variation in the $O-C$ diagram of U Gem with a semiamplitude of 0.0008 days with a period of 8 yr (i.e., $\Delta P/P = 1.7 \times 10^{-6}$). For U Gem we take $q = 0.5$ and $M_2 = 0.6\ M_\odot$ (Martin 1988), which, for a lobe-filling star, implies that $R_2/D = 0.32$ (Eggleton 1983). We note that $f = 0.34$ for the $0.6\ M_\odot$ main-sequence star tabulated in Schwarzschild (1958) and Plavec (1960) gives $K = 0.0565$. For model B this gives $\lambda = -0.9$ (Fig. 2). Thus in this case we find that the required radius change is $\Delta R_2/R_2 = 9 \times 10^{-5}$. Taking $R_2 = 0.6\ R_\odot$ and $L_2 = 0.2\ L_\odot$, we find that the shortest time scale on which such a change can be achieved is ≈ 1800 yr.

b) V471 Tauri

The system parameters given by Applegate and Patterson (1987) are $M_1 = 0.7\ M_\odot$, $M_2 = 0.8\ M_\odot$, $R_2/D = 0.25$, and $P = 12.5$ hr. For a $0.8\ M_\odot$ main-sequence star, Cisneros-Parra (1970) gives $K = 0.031$. If we take $f = 0.1$ (Applegate and Patterson 1987) and use model B as being more appropriate to a K2 V star so that $\lambda = -2.6$ we find

$$\frac{\Delta P}{P} = -7.6 \times 10^{-4} \frac{\Delta R_2}{R_2}. \quad (9)$$

Skillman and Patterson (1988) interpret the $O-C$ diagram of V471 Tau in the period 1969–1974 as showing a period decrease $\Delta P/P = -2 \times 10^{-6}$ which occurs over a time scale of 4 yr. Such a period change implies, using equation (9), a radius change of $\Delta R_2/R_2 = 2.6 \times 10^{-3}$.

Taking $R_2 = 0.8\ R_\odot$, and $L_2 = 0.5\ L_\odot$ as suitable values for a K2 V star we see from equations (7) and (8) that such a change can be achieved only on a time scale of at least 8300 yr. We conclude, in agreement with Applegate and Patterson, that tidal distortion is unlikely to be able to produce the period variation seen in V471 Tau. The longer time scale of V471 Tau in comparison to U Gem is largely a result of its smaller ratio of R_2/D.

V. ANISOTROPIC MAGNETIC PRESSURE

To provide a quadrupole distortion, Applegate and Patterson posit a change in the pressure at the base of the convective zone caused by increased magnetic pressure. This cuts out the intermediate step of tidal distortion, but requires that the magnetic pressure distort the star directly. In comparison to equations (6) and (7) in their paper we obtain the following. For the stellar parameters given above, and taking $k = 0.15$, corresponding to the bipolytrope we find that to effect a period change of $\Delta P/P = 2 \times 10^{-6}$ requires

$$\frac{\Delta Q}{I} = 2.4 \times 10^{-5}. \quad (10)$$

Again, by considering small changes computed in our models, and regarding the change of polytropic constant as corresponding to an addition of magnetic pressure, we find

$$\Delta I = 3I \left(\frac{\Delta M}{M}\right)_{CZ} \left(\frac{B^2}{8\pi P}\right)_{CZ}. \quad (11)$$

Here ΔM is the mass in the convection zone, B is the magnetic field, P the pressure, and the subscript CZ applies to the base of the convection zone. Of course for the spherical models that we compute $\Delta Q = Q = 0$. However, if the magnetic field induces a maximum amount of anisotropy (e.g., elongation or contraction along one axis), we would expect $\Delta Q \approx \Delta I$. Combining these expressions and with $\Delta M/M = f = 0.1$, we obtain $B^2/8\pi P = 8 \times 10^{-5}$ at the base of the convection zone. The energy contained in this magnetic field is

$$E_{MAG} = \int_{CZ} P \left(\frac{B^2}{8\pi P}\right) dV, \quad (12)$$

and integrating Applegate and Patterson's equations (12) and (13) over the convection zone depth $\Delta R \approx R/3$, we find $E_{MAG} \approx 4 \times 10^{42}$ ergs. The time scale on which this can be provided is $E_{MG}/L \approx 60$ yr, assuming that all the stellar energy output is channeled into field generation.

VI. CONCLUSIONS

We have considered the two mechanisms for changing the quadrupole moment of a star in a binary system, and hence the orbital period, put forward by Applegate and Patterson (1987) and Warner (1988).

In each paper a magnetic field is used to alter the structure of the star. Applegate and Patterson suppose that the magnetic field distorts the star directly, whereas Warner includes tidal distortion to link a change in the size of a star to its quadrupole moment.

We estimate the amount of energy required to effect the proposed changes and find that it is too large to be supplied on the time scales observed. Of the two methods, the direct distortion of Applegate and Patterson (1987) comes closest to the observations since it avoids the intermediate step of tidal coupling favored by Warner (1988). However, even in this most favorable case, our estimate of the shortest time for change is 15 times the observed time scale. Thus unless the magnetic field configuration can cycle between isotropic and anisotropic configurations without requiring a net input of energy, the rate at which energy can be supplied in order to effect the observed changes is too small.

To the extent that they are simplified, our calculations are not conclusive. At the same time, however, they are based on simple physics and it is hard to see that they can be one to two orders of magnitude incorrect as is required to match the observations. The uncertainties in the radius variations (Gilliland 1982) come about because only a small amount of material need be involved to change the radius significantly. In contrast, changes of the stellar quadrupole moment require the involvement of a significant amount of the stellar material and thus require significant amounts of energy to effect. We conclude that in general the causes of the period changes in these close binary systems must be looked for elsewhere (e.g., Bois *et al.* 1988; Africano *et al.* 1978).

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