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Are overcontact binaries undergoing thermal relaxation oscillation with variable angular momentum loss?

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ABSTRACT

Orbital period variations of five W-type overcontact binaries, GW Cep, VY Cet, V700 Cyg, EM Lac and AW Vir, are presented based on the analysis of all available times of light minimum. It is discovered that the period of GW Cep is decreasing at a rate of $\mathrm{d}P/\mathrm{d}t = -6.62 \times 10^{-8}$ d yr^{-1}. For VY Cet and V700 Cyg, a cyclic oscillation is found superimposed on a secular period increase, which can be explained either by the light-time effect of an assumed third body or by magnetic activity cycles. For the other two, EM Lac and AW Vir, the periods show a secular increase. GW Cep is a low mass ratio system with $q = 0.37$, while the others are high mass ratio systems ($q = 0.67$, 0.65, 0.63 and 0.76, respectively). The period changes of the five sample stars are in good agreement with Qian's conclusion that low mass ratio overcontact binaries usually show a decreasing period, while the periods of high mass ratio systems are increasing.

Based on the period variations of 59 overcontact binaries, a statistical investigation of period change is given. It is confirmed that the period change of a W UMa-type binary star is correlated with the mass ratio (q) and with the mass of the primary component (M_1). Meanwhile, some statistical relations (M_1–P, J_s–M_1, J_s–M_2 and J_s–P) for overcontact binaries are presented using the absolute parameters of 78 systems. From these relations, the following results may be drawn: (i) free mass transfer in both directions exists between the components, which is assumed by thermal relaxation oscillation (TRO) theory; (ii) angular momentum loss (AML) can make a W UMa-type star maintain shallow overcontact and not evolve from overcontact to semidetached configurations as proposed by Rahunen; (iii) the evolution of the W UMa-type systems may be oscillation around a critical mass ratio, while the critical mass ratio varies with the mass of the primary component. These results can be plausibly explained by the combination of the TRO and the variable AML via a change of depth of the overcontact, which is consistent with the X-ray and *IUE* observations.

Key words: binaries: close – stars: evolution – stars: magnetic fields – stars: mass-loss.

1 INTRODUCTION

Although more than 30 years have passed since Lucy's first model (Lucy 1968), W UMa-type overcontact binaries still represented a challenge in astrophysics. Such systems have been investigated by many papers in order to understand their structure and evolutionary state. Most studies are based on Lucy's (1968) common convective envelope (CCE) model, which satisfactorily explains the almost equal minima characteristic of the light curves of these systems. This is caused by large-scale energy transfer from the primary to the secondary component, roughly equalizing surface temperatures over the entire system. Lucy's model, based on the assumption of thermal equilibrium, resulted from difficulties in explaining Eggen's (1967) observed period–colour relation for W UMa stars. After Lucy's (1968) pioneering work, several equilibrium models were suggested by Biermann & Thomas (1972), Whelan (1972) and Shu, Lubow & Anderson (1976). However, these models fail to understand the observed light curve (Kähler 1989), and as pointed out by Kähler (1997) thermal equilibrium in late-type overcontact binaries, whether evolved or not, usually cannot be established.

Since the difficulties in construction equilibrium models and considering the existence of very noticeable fluctuation in the light curve and period changes of those binary stars, several authors have proposed thermal non-equilibrium models (e.g. Lucy 1976; Flannery 1976; Robertson & Eggleton 1977). These models assumed that the components of an overcontact binary were not in thermal equilibrium and can exchange matter freely between themselves. When total mass and angular momentum are conserved, they

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predicted that the systems must undergo cycles around the state of marginal overcontact, i.e. the thermal relaxation oscillation (TRO) state. TRO models can explain the shallow overcontact feature observed for many W UMa-type stars and predict that parameters, such as the separation, the mass ratio and the orbital period, should vary on thermal time-scales and, indeed, secular period changes (both increasing and decreasing) having time-scales of the right order of magnitude are observed. However, as discussed by Rahunen (1981), Smith (1984) and Rucinski (1985a), the main problem of TRO remaining is the apparent non-existence of short-period, broken-overcontact binaries.

In order to avoid the difficulty with the TRO model, Rahunen (1981) dropped the usual assumption of conservation of angular momentum. He treated the orbital angular momentum as a free parameter, keeping the system in marginal overcontact at each time-step. His study showed that overcontact binaries can keep in shallow overcontact by assuming a hypothetical angular momentum loss (AML) rate d $\ln j/\mathrm{d}t \sim 2 \times 10^{-9}$ yr^{-1}. Too large an AML rate can cause the system to quickly reach the L_2-surface and it may finally coalesce into a single star. While a smaller AML rate than the critical rate cannot maintain a system in overcontact and the system oscillates again. As pointed by Rahunen (1981) and by Smith (1984), this model leaves two questions unsolved: what is the mechanism of AML and what mechanism keeps the system permanently in shallow overcontact?

Recently, the studies by Qian (2001a,b) have indicated that the period change of a W UMa-type binary system may be related to the mass ratio and the mass of the primary component. There may exist a critical mass ratio (q_{cr}) for the period change of a W UMa-type binary star, systems with $q > q_{\mathrm{cr}}$ usually show an increasing period, while the periods of low mass ratio systems ($q < q_{\mathrm{cr}}$) are decreasing. Moreover, the critical mass ratio is correlated with M_1. These observational findings may suggest that AML can keep W UMa-type stars in shallow overcontact. By the combination of TRO and variable AML, W UMa-type stars may oscillate around a critical mass ratio. In the present paper, orbital period changes of five overcontact binary stars are presented and the period changes for this type of star are studied statistically. Then, based on some statistical relations and on the X-ray and the extreme ultraviolet (EUV) data, the evolutionary model, TRO plus variable AML via the change of depth of overcontact, is discussed. The physical properties of the five W UMa-type binary stars are shown in Table 1.

2 PERIOD CHANGES OF THE RESPECTIVE STARS

2.1 GW Cephei

Light variability of the short-period eclipsing binary star, GW Cep (=CSV 5941 = BV 7), was discovered by Strohmeier (Geyer, Kippenhahn & Strohmeier 1955). The first photoelectric light curve was published by Meinunger & Wenzel (1965), who found that the system is a typical EW-type binary system (whose light curve shows a continuous variation and has two nearly equal minima) and determined the spectral type to be G3. Later, photoelectric light curves and solutions using the Russell–Merrill method were published by Hoffman (1982) who also derived the improved linear ephemeris:

$$\mathrm{MinI} = 2438651.545 + 0.31884945E. \quad (1)$$

Combined with Hoffman's photoelectric time of light minimum, the following light elements were obtained by Landolt (1992):

$$\mathrm{MinI} = 2438651.5445 + 0.318851065E. \quad (2)$$

Kaluzny (1984) determined photometric solutions of GW Cep using Rucinski's code. Recently, absolute parameters of the W-type overcontact binary were published by Maceroni & van't Veer (1996).

Although, light elements of GW Cep were given by several authors (e.g. Hoffman 1982; Landolt 1992), the system was neglected for periodicity studies. In order to study the period variation of the system, all the available times of light minimum were collected and are listed in Table 2. 42 visual or photographic timings published in BBSAG Bulletins have been collected in the Eclipsing Binaries Minimum Data base (hereafter EBMD) (available at http:/nac.oa.uj.pl/ktt/ktt.html). During the computation of the $O - C$ values, a problem in cycle (E) count is encountered. When Landolt (1992) ephemeris was used, the $O - C$ values of the last five photoelectric or charge-coupled device (CCD) minima show a strange variation. The scatters of those values are larger than $P_{\mathrm{e}}/2$ (0.1594 d) and it is very difficult to determine the correct values of E. This may indicate that the period given by Landolt needs to be improved. If the value of orbital period in an ephemeris is not correct, the accumulative effect can cause the $O - C$ to be larger than $P_{\mathrm{e}}/2$ and one cannot correctly calculate the values of E and $O - C$. Based on all photoelectric and CCD times of light minimum, a new linear ephemeris

$$\mathrm{MinI} = 2448544.8704 + 0.318830197E, \quad (3)$$

is derived. Comparing this ephemeris with Landolt's, a period difference $\Delta P = -2.09 \times 10^{-5}$ d is obtained. If the binary revolves 40 000 cycles, the accumulation can cause a standard change $\Delta(O - C) = -0.8347$ d $= -2.6P_{\mathrm{e}}$ in the $O - C$ values and result in an incorrect computation of E and $O - C$ values.

The $O - C$ values, based on the new ephemeris (equation 3), are computed and are listed in Table 2. The corresponding $O - C$ curve is plotted graphically against epoch number in Fig. 1, where open circles refer to visual or photographic data and dots to photoelectric or CCD observations. As shown in Fig. 1, although the $O - C$ values show large deviations, the general $O - C$ trend indicates that the orbital period of GW Cep is variable. Considering that all the $O - C$ data have a parabolic variation, and using the weight 1 for visual or photographic observations and weight 8 for photoelectric or CCD data, the following quadratic ephemeris with the mean error for each term

$$\mathrm{MinI} = 2446589.8721(3) + 0.318830285(23)E - 2.89(9) \times 10^{-11}E^2, \quad (4)$$

is obtained. The coefficient of the square term indicates a continuous period decrease with a rate of $\mathrm{d}P/\mathrm{d}t = -6.62 \times 10^{-8}$ d yr^{-1}.

2.2 VY Ceti

The eclipsing character of VY Cet was discovered by Hoffmeister (1948) who observed the system photographically. The first

Table 1. Physical properties of eight W-type overcontact binaries.

Star name	M_1	M_2	R_1	R_2	P	i	q	Ref.
GW Cep	1.06	0.39	1.05	0.67	0.3188	83°.9	0.37	(1)
VY Cet	1.02	0.68	1.01	0.83	0.3408	78°.4	0.67	(1)
V700 Cyg	0.92	0.60	1.04	0.86	0.3400	79°.9	0.65	(2)
EM Lac	1.06	0.67	1.19	0.97	0.3891	74°.4	0.63	(1)
AW Vir	1.11	0.84	1.08	0.95	0.3540	81°.2	0.76	(2)

References: (1) Maceroni & van't Veer (1996); (2) Niarchos et al. (1997).

Table 2. Times of light minimum for GW Cep.

JD Hel.	Min.	Method	E	$O-C$	Ref.
2400000+					
38383.711	I	pe	−31870	−0.0410	(1)
38462.474	I	pe	−31623	−0.0291	(1)
38651.545	I	pe	−31030	−0.0244	(1)
38652.503	I	pe	−31027	−0.0229	(1)
42623.393	II	pv	−18572.5	−0.0036	(2)
42633.422	I	pv	−18541	−0.0177	(2)
43016.353	I	pv	−17340	−0.0018	(2)
44200.48205	I	pe	−13626	−0.0028	(3)
47823.353	I	pv	−2263	−0.0047	(2)
47857.317	II	pv	−2156.5	+0.0039	(2)
47922.346	II	pv	−1952.5	−0.0084	(2)
47970.341	I	pv	−1802	+0.0026	(2)
48125.450	II	pv	−1315.5	+0.0007	(2)
48146.339	I	pv	−1250	+0.0063	(2)
48167.384	I	pv	−1184	+0.0086	(2)
48174.400	I	pv	−1162	+0.0103	(2)
48187.299	II	pv	−1121.5	−0.0033	(2)
48447.478	II	pv	−305.5	+0.0102	(2)
48467.403	I	pv	−243	+0.0083	(2)
48486.362	II	pv	−183.5	−0.0031	(2)
48489.390	I	pv	−174	−0.0039	(2)
48490.352	I	pv	−171	+0.0016	(2)
48492.433	II	pv	−164.5	+0.0102	(2)
48495.448	I	pv	−155	−0.0037	(2)
48496.414	I	pv	−152	+0.0058	(2)
48497.363	I	pv	−149	−0.0017	(2)
48499.436	II	pv	−142.5	−0.0011	(2)
48500.396	II	pv	−139.5	+0.0024	(2)
48503.430	I	pv	−130	+0.0075	(2)
48517.455	I	pv	−86	+0.0040	(2)
48524.311	II	pv	−64.5	+0.0051	(2)
48544.87103	I	pe	0	+0.0006	(4)
48573.421	II	pv	89.5	+0.0153	(2)
48628.413	I	pv	262	+0.0091	(2)
48892.404	I	pv	1090	+0.0087	(2)
49024.400	I	pv	1504	+0.0090	(2)
49592.5452	I	pe	3286	−0.0012	(5)
50283.447	I	pv	5453	−0.0045	(2)
50285.380	I	pv	5459	+0.0156	(2)
50299.392	I	pv	5503	−0.0010	(2)
50313.411	I	pv	5547	−0.0105	(2)
50357.416	I	pv	5685	−0.0041	(2)
50380.369	I	pv	5757	−0.0068	(2)
50423.257	II	pv	5891.5	−0.0015	(2)
50673.387	I	pv	6674	+0.0062	(2)
50684.379	II	pv	6710.5	−0.0014	(2)
50688.368	I	pv	6723	+0.0022	(2)
51391.5409	II	pe	8928.5	−0.0049	(6)
51622.5520	I	ccd	9653	+0.0137	(7)
51634.80913	II	ccd	9691.5	−0.0041	(7)
51968.3054	II	pe	10737.5	−0.0042	(8)

References in Table 2:
(1) Meinunger & Wenzel (1965); (2) refer to times of light minimum collected at the EBMD; (3) Hoffman (1982); (4) Landolt (1992); (5) Agerer & Hubscher (1995); (6) Agerer & Hubscher (2001); (7) Nelson (2001); (8) Pribulla et al. (2001).

photoelectric light curves in U, B and V were obtained by Lapasset & Claria (1986). They published two times of primary minimum light and two of the secondary minimum light, and derived a linear ephemeris. Based on their observations, Lapasset & Claria (1986) obtained the photometric solutions of the binary star; it is shown that VY Cet is a W-type overcontact binary. Recently, absolute parameters were derived by Maceroni & van't Veer (1996).

At the EBMD, 244 times of light minimum have been collected. Unfortunately, of the 244 timings, only four are photoelectric data published by Lapasset & Claria (1986). The $O-C$ curve based on the linear ephemeris given by Lapasset & Claria (1986),

$$\mathrm{Min I} = 2445284.7219 + 0.3408087E, \tag{5}$$

is shown in Fig. 2 where solid dots correspond to the photoelectric observations and open circles correspond to visual or photographic data. Although the data shown has a slight scatter, the general $O-C$ trend indicates that the orbital period of the systems is variable and its change may be complex. Since a sinusoidal variation may exist in Fig. 2, a sinusoidal term is added to a quadratic ephemeris to obtain a good fit to the observations. With the weights set as

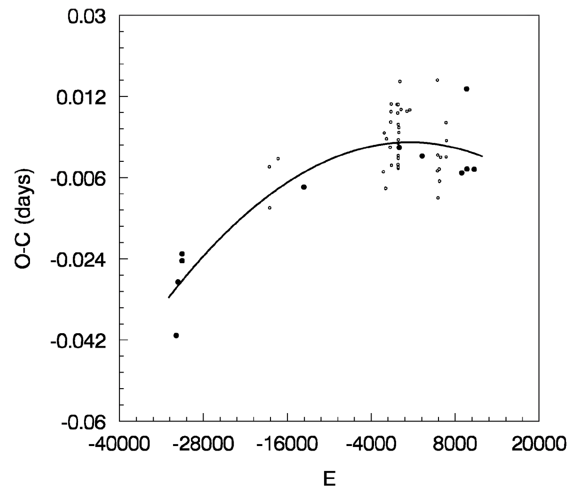

Figure 1. $O-C$ plot in days for the W-type overcontact binary GW Cep. Circles refer to visual or photographic observations and dots to photoelectric or CCD data. Also given in the solid line is its description by a quadratic ephemeris (equation 4).

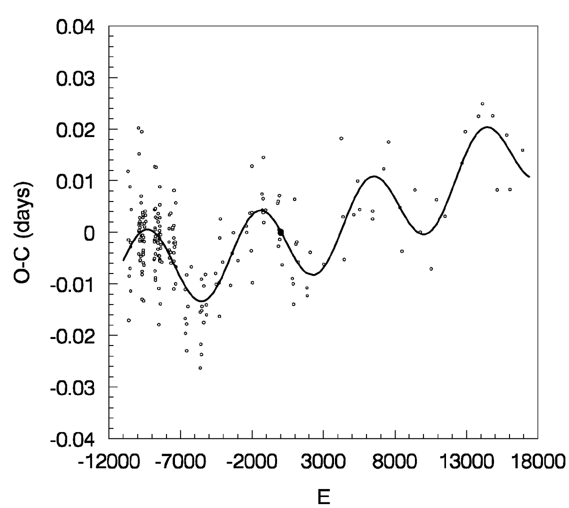

Figure 2. $O-C$ curve of VY Cet from the linear ephemeris equation (5). Symbols are the same as in Fig. 1. The solid line refers to its description by equation (6).

Table 3. Parameters of the assumed third bodies in VY Cet and V700 Cyg.

Parameters	VY Cet	V700 Cyg	Units
A	0.0077(±0.0007)	0.0095(±0.0020)	days
T	7.3(assumed)	39.8(assumed)	years
$a'_{12}\sin i'$	1.33(±0.12)	1.65(±0.35)	au
$f(m)$	$4.5(\pm1.2)\times10^{-2}$	$2.8(\pm1.8)\times10^{-3}$	$M_\odot$
$m_3(i'=90°)$	0.62(±0.12)	0.20(±0.08)	$M_\odot$
$m_3(i'=70°)$	0.67(±0.12)	0.22(±0.08)	$M_\odot$
$m_3(i'=50°)$	0.87(±0.14)	0.27(±0.09)	$M_\odot$
$m_3(i'=30°)$	1.56(±0.24)	0.44(±0.12)	$M_\odot$
$m_3(i'=10°)$	11.2(±2.47)	1.81(±0.16)	$M_\odot$
$a_3(i'=90°)$	4.34(±0.89)	14.7(±6.4)	au
$a_3(i'=70°)$	4.19(±0.83)	14.3(±5.9)	au
$a_3(i'=50°)$	3.77(±0.69)	13.3(±5.0)	au
$a_3(i'=30°)$	3.04(±0.54)	11.8(±4.1)	au
$a_3(i'=10°)$	1.16(±0.28)	7.98(±3.1)	au

those used for GW Cep, a weighted least-squares solution yields the following ephemeris:

$$\mathrm{MinI} = 2445284.7194(9) + 0.34080942(7)E + 2.4(9) \times 10^{-11}E^2 + 0.0077(7)\sin[0\overset{\circ}{.}0459E + 160\overset{\circ}{.}1(1\overset{\circ}{.}8)]. \quad (6)$$

With the quadratic term of this equation, a period increase rate, $\mathrm{d}P/\mathrm{d}t = +5.16\times10^{-8}$ d yr^{-1}, is determined.

The quadratic part of equation (6) suggests a periodic variation with an amplitude of $A = 0.0077$ d and a period of $T = 7.3$ yr. If the small-amplitude oscillation in the period of VY Cet is caused by the presence of a third body, then assuming that the orbit of the third body is circular, a mass function $f(m) = 4.5(\pm1.2)\times10^{-2}$ is obtained with the well-known equation:

$$f(m) = \frac{(M_3\sin i')^3}{(M_1+M_2+M_3)^2} = \frac{4\pi^2}{GP'^2}(a_{12}\sin i')^3, \quad (7)$$

where P' and i' are the period and the orbital inclination of the third body, respectively. With the absolute parameters listed in Table 1, the values of the masses and the orbital dimensions of the third body for several different values of i' are listed in Table 3. If the additional companion is coplanar to the orbit of the binary pair (i.e. $i' = 79\overset{\circ}{.}8$), the mass of the third body will be $M_3 = 0.63\ M_\odot$.

2.3 V700 Cygni

The orbital period change of the W-type overcontact binary, V700 Cyg, was studied by Niarchos, Hoffmann & Duerbeck (1997). After their study, two photoelectric times of light minimum, 49999.3024 and 49999.4474, have been published by Agerer & Hubscher (1999). One CCD eclipse time, HJD 245 0324.371, has been given in BBSAG Bulletin 113. The $O-C$ curve calculated with the ephemeris: MinI = 244 5163.4896 + 0.340 045 602E shows rapid variation, which was attributed to short-term instability by Niarchos et al. (1997). The difference of the cycles of the two timings published by Agerer & Hubscher (1999) is only $\Delta E = 0.5$, but the difference of the corresponding $O-C$ values is $\Delta(O-C) = -0.025$. This may indicate that the ephemeris period (0.340 045 602 d) is too much larger than the real period, and it should be reduced by 0.05 d, i.e. the period of the system is about $P = 0.29$ d. The rapid jumps in the $O-C$ curve of Niarchos et al. (1997) are caused by the miscalculation of E and $O-C$ values via an incorrect ephemeris period. The situation is the same as those of GW Cep and VW Boo (Qian

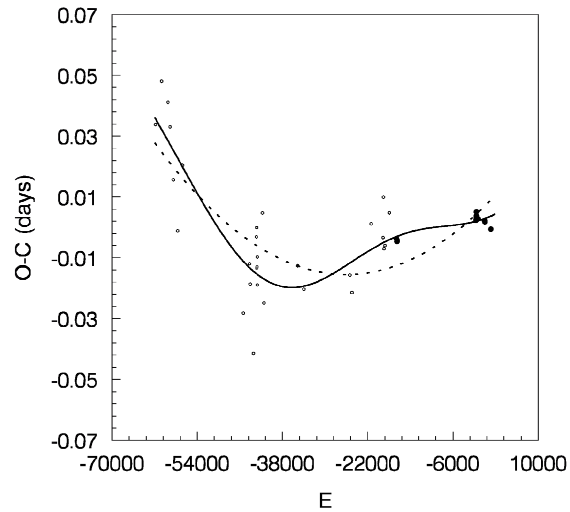

Figure 3. $O-C$ curve of V700 Cyg computed with the linear ephemeris (equation 8). Symbols are the same as in Figs 1 and 2. The dashed line refers to the quadratic part of equation (9).

& Zhu 2002). Based on all available times of light minimum, a new linear ephemeris

$$\mathrm{MinI} = 2449999.3003(52) + 0.29063148(14)E \quad (8)$$

is derived.

The $O-C$ plot based on the new ephemeris is shown in Fig. 3. With weights as those used for GW Cep and VY Cet, a weighted least-squares solution leads to the following ephemeris:

$$\mathrm{MinI} = 2449999.3073(39) + 0.29063321(35)E + 3.34(58) \times 10^{-11}E^2 + 0.0095(20)\sin[0\overset{\circ}{.}0072\times E + 205\overset{\circ}{.}5(4\overset{\circ}{.}7)], \quad (9)$$

which can give a good description to the general trend of the $O-C$ curve (solid line in Fig. 3). The quadratic part of the ephemeris suggests a continuous period increase at a rate of $\mathrm{d}P/\mathrm{d}t = +8.41\times10^{-8}$ d yr^{-1}, and the periodic term in equation (9) indicates that a possible sinusoidal variation is superimposed on the secular increase. As in the case of VY Cet, if the cyclic variation is caused by the light-time effect of a third body, the parameters of the assumed third body are computed and are shown the Table 3. If the orbit plan of the third body is parallel to the visual line, the mass of the third body should be $m_3 = 0.2\ M_\odot$. In this case, the supposed third bodies are difficult to detect because its luminosity is extremely small.

2.4 EM Lacertae

The W-type overcontact binary, EM Lac, was discovered to be a variable by Kurockin (1945). Later, Kukarkina (1953) derived a normal light curve and the light elements

$$\mathrm{MinI} = 2432797.285 + 0.388924E, \quad (10)$$

which is reported in the third edition of the General Catalogue of Variable Stars. The eclipse time, HJD 241 7793.438, was not represented satisfactorily with the above linear ephemeris, so Kukarkina suspected there may be a variation of the period. However, since the assumed period change is based on the single photographic timing and the time intervals between this timing and the others are very large, the period variation may not be true. Romano & Perissinotto (1966) determined three times of minimum light and found they are in good agreement with the above elements. The first photoelectric light curves in B and V have been published by Broglia & Conconi (1974) who collected many times of light minimum of the system and obtained linear light elements:

$$\mathrm{MinI} = 2438259.5444 + 0.38913342E. \tag{11}$$

Broglia & Conconi's light curves were analysed by Maceroni et al. (1983) using the Wilson–Devinney method, and the absolute parameters were derived.

After the collection of Broglia & Conconi (1974), five visual or photographic timings published in BBSAG Bulletins have been compiled at the EBMD, and one photoelectric and one CCD timing have been published by Agerer & Hubscher (1996) and by Nelson (2001), respectively. The $O-C$ diagram, based on the linear ephemeris of Broglia & Conconi (1974), is plotted in Fig. 4. Since the time intervals between the first photographic timing (HJD 241 7793.438) and the others are very large, the $O-C$ value of this timing is not shown in the figure. As displayed in this figure, although the visual or the photographic timings show large scatter, the general trend of the $O-C$ data may suggest that the period change of EM Lac is a continuous increase. With the same weights as those used for the former systems, a least-squares solution yields the following quadratic ephemeris:

$$\begin{aligned}\mathrm{MinI} = {} & 2438259.5446(1) + 0.38913311(1)E \\ & + 4.84(14) \times 10^{-11} E^2\end{aligned} \tag{12}$$

and a period increase rate of $\mathrm{d}P/\mathrm{d}t = +9.09 \times 10^{-8}$ d yr^{-1}. If only the photoelectric and CCD times of light minimum are used, the quadratic ephemeris

$$\begin{aligned}\mathrm{MinI} = {} & 2438259.54489(1) + 0.38913288(1)E \\ & + 5.361(2) \times 10^{-11} E^2\end{aligned} \tag{13}$$

is obtained. As we can see in Fig. 5, the ephemeris can described the data well (solid line in the figure). With this ephemeris, a period increase rate $\mathrm{d}P/\mathrm{d}t = +10.7 \times 10^{-8}$ d yr^{-1} is derived, which is nearly equal to that obtained using all the data. This may indicate that the period increase is true.

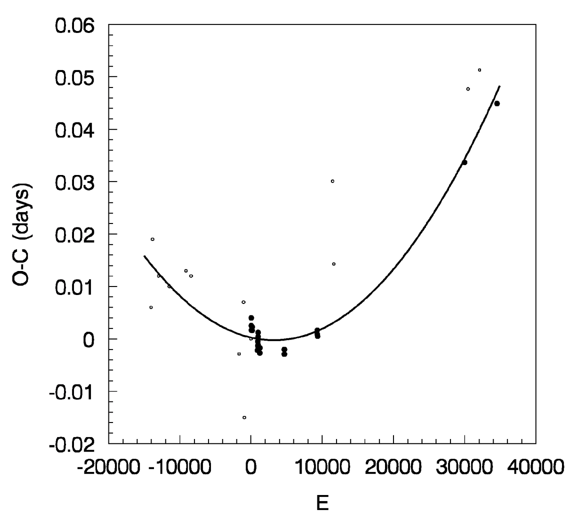

Figure 4. $O-C$ values in days for EM Lac and its description by a quadratic ephemeris (solid line).

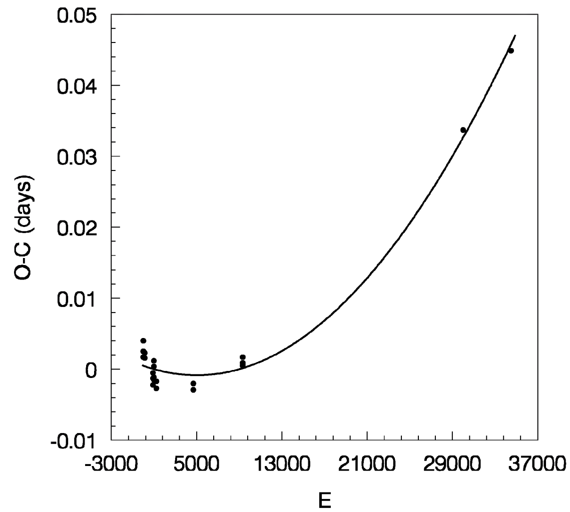

Figure 5. $O-C$ curve of EM Lac based on all photoelectric or CCD data.

2.5 AW Virginis

As in the case of V700 Cyg, the orbital period of AW Vir has recently been studied by Niarchos et al. (1997). After their study, 14 times of light minimum have been collected at the EBMD and published by Agerer, Dahm & Hubscher (1999) and by Agerer & Hubscher (2000). The $O-C$ values of those times of light minimum, not compiled by Niarchos et al. (1997) or published after their study, are computed with the following ephemeris:

$$\mathrm{MinI} = 2445022.6496 + 0.35399707E. \tag{14}$$

The corresponding $O-C$ curve is displayed in Fig. 6. With a weighted least-squares method, the following quadratic ephemeris

$$\begin{aligned}\mathrm{MinI} = {} & 2445022.6477(4) + 0.35399735(1)E \\ & + 6.74(13) \times 10^{-12} E^2\end{aligned} \tag{15}$$

and a continuous period increase at rate of $\mathrm{d}P/\mathrm{d}t = +1.39 \times 10^{-8}$ d yr^{-1} are derived. However, as seen in Fig. 6, the $O-C$ data, especially those of visual or photographic observations, show very large scatter. In order to check the period change of the binary system, more photoelectric or CCD timings are needed.

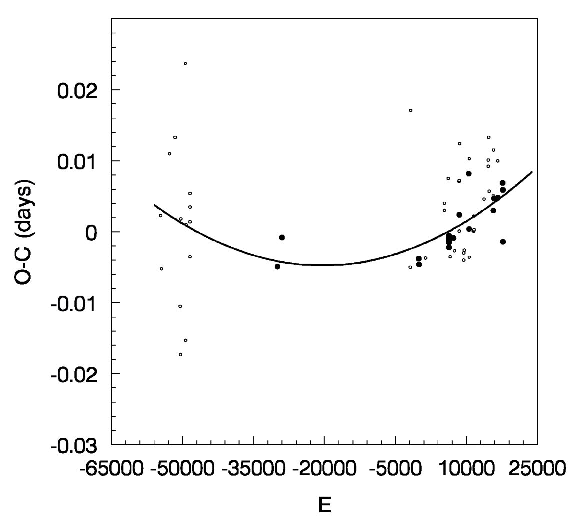

Figure 6. The same as in Fig. 4, but for AW Vir.

3 STATISTICAL INVESTIGATION OF PERIOD CHANGE FOR AN OVERCONTACT BINARY SYSTEM

In previous subsections, the rates of secular period changes of five W-type overcontact binaries, GW Cep, VY Cet, V700 Cyg, EM Lac and AW Vir, have been determined. The rates of period changes of the five W UMa stars are listed in the fifth column of Table 4 in the order of decreasing mass ratio. Also shown in the table are the rates of period changes of two another overcontact binaries, CC Com and V523 Cas, with the shortest mass and period among W UMa-type binaries, published by Qian (2001c) and by Samec et al. (2001), respectively. Those shown in the sixth column are the time-scales of the corresponding period changes in units of years. This table shows that the orbital periods of the high mass ratio systems (AW Vir, VY Cet, V700 Cyg, EM Lac and V523 Cas) are increasing, while the low mass ratio system, CC Com and GW Cep, shows a decreasing period. The period changes of these systems are in good agreement with the result from Qian (2001a) that the period of a W UMa-type binary star is correlated with the mass ratio (q).

The status of the period change can be illustrated in Fig. 7 where the positions of 59 W UMa-type binary stars showing secular period change are plotted in the q–M_1 diagram. Apart from the systems listed in Table 4, the period changes of various sample stars are from the compilations by Qian (2001a,b). The values of q and M_1 of many sample stars were given by Maceroni & van't Veer (1996). For some systems such as UV Lyn, BB Peg, UZ Leo, XZ Leo and others, the absolute parameters are directly from or revised with the spectroscopic elements published by Lu & Rucinski (1999) and

Table 4. The rates of period change for some overcontact binaries.

Star name	Type	P	q	$\mathrm{d}P/\mathrm{d}t$ (days/year)	$\lvert P/\mathrm{d}P/\mathrm{d}t \rvert$ (yr)
AW Vir	W	0.3540	0.76	$+1.39 \times 10^{-8}$	2.55×10^{7}
VY Cet	W	0.3408	0.67	$+5.16 \times 10^{-8}$	6.60×10^{6}
V700 Cyg	W	0.2906	0.65	$+8.41 \times 10^{-8}$	3.46×10^{6}
EM Lac	W	0.3891	0.63	$+9.09 \times 10^{-8}$	4.28×10^{6}
V523 Cas	W	0.2337	0.57	$+3.19 \times 10^{-8}$	7.33×10^{6}
CC Com	W	0.2207	0.52	-4.39×10^{-8}	5.03×10^{6}
GW Cep	W	0.3188	0.37	-6.62×10^{-8}	4.82×10^{6}

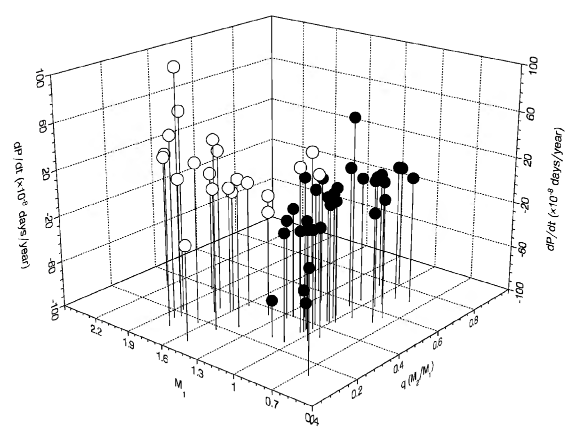

Figure 7. Positions of observed overcontact binaries in the q–M_1 diagram. Solid dots represent the cooler systems ($M_1 < 1.35\ \mathrm{M}_\odot$) and open circles denote the hotter ones. All the sample stars are shifted from the xy plane by a distance corresponding to the value of $\mathrm{d}P/\mathrm{d}t$ (in 10^{-8} d), plotted on the vertical (z) axis.

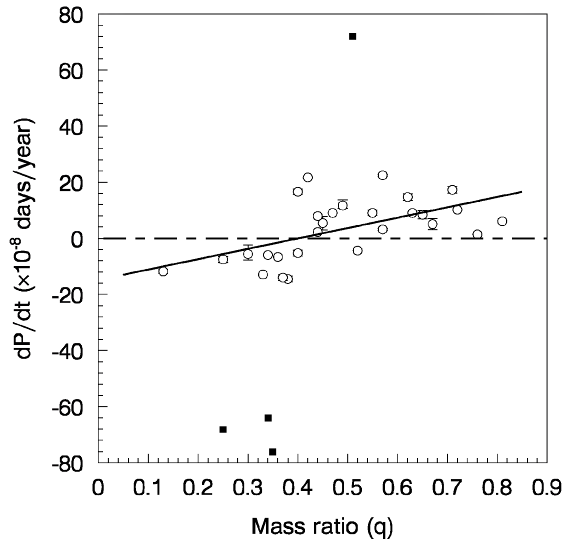

Figure 8. A possible relation between $\mathrm{d}P/\mathrm{d}t$ and q for cooler overcontact binaries. See the text for details.

Rucinski & Lu (1999). For the other systems (e.g. BF Pav, EZ Hya and V366 Cas) only the photometric solutions were published and the values of M_1 are estimated using the spectral type. In Fig. 7 it is shown that only cooler systems with a low mass ratio show a decreasing period.

For the cooler systems ($M_1 < 1.35\ \mathrm{M}_\odot$), there may exist a correlation between the period change $\mathrm{d}P/\mathrm{d}t$ and the mass ratio q, which was clearly seen in Fig. 8 where $\mathrm{d}P/\mathrm{d}t$ is plotted graphically against q. The value of the $\mathrm{d}P/\mathrm{d}t$ error has been determined with the error of the coefficient of the quadratic term in the ephemeris for each system. The corresponding error bar is shown in Fig. 8. However, for some systems, the error bars are too small to be seen in this figure. Four systems, CE Leo, VW Cep, LT Pav and EZ Hya, showed rather large rates of period change when compared with the other normal systems (displayed as solid squares). For these four systems, the periods need further investigatation. Based on the rates of the other binary stars, a possible relation between $\mathrm{d}P/\mathrm{d}t$ and q

$$\mathrm{d}P/\mathrm{d}t = -1.5(\pm 0.8) \times 10^{-7} + 3.7(\pm 1.7) \times 10^{-7} q \tag{16}$$

is derived, where $\mathrm{d}P/\mathrm{d}t$ is in units of 10^{-7} d yr^{-1}. This relation is shown in Fig. 8 (solid line), which tells us that when $q = 0.41$, $\mathrm{d}P/\mathrm{d}t = 0\ (\pm 1.1)$. It is predicted from equation (16) that the periods of systems with $q > 0.41$ will be increasing, while the low mass ratio systems ($q < 0.41$) show a secular period that is decreasing.

It can also be seen from Fig. 7 that the period change may be correlated with the mass of the primary component (M_1). For the low mass ratio systems ($q < 0.4$), a connection between $\mathrm{d}P/\mathrm{d}t$ and M_1 may exist that is more clearly seen in Fig. 9 where $\mathrm{d}P/\mathrm{d}t$ is displayed against M_1. Since the error bars for most of the systems are too small to be seen, they are not shown in this figure. With the same method as that used for equation (16), a possible correlation between $\mathrm{d}P/\mathrm{d}t$ and M_1

$$\mathrm{d}P/\mathrm{d}t = -8.4(\pm 2.9) \times 10^{-7} + 6.4(\pm 2.0) \times 10^{-7} M_1 \tag{17}$$

is obtained, where $\mathrm{d}P/\mathrm{d}t$ is in unit of 10^{-7} d yr^{-1} and M_1 has units of solar mass. This relation indicates that the larger M_1 is, the larger $\mathrm{d}P/\mathrm{d}t$ will be. When $M_1 > 1.31$, $\mathrm{d}P/\mathrm{d}t > 0\ (\pm 3.9)$. As pointed out by Qian (2001b), this relation predicts that, for hotter overcontact binaries, the period should be increasing.

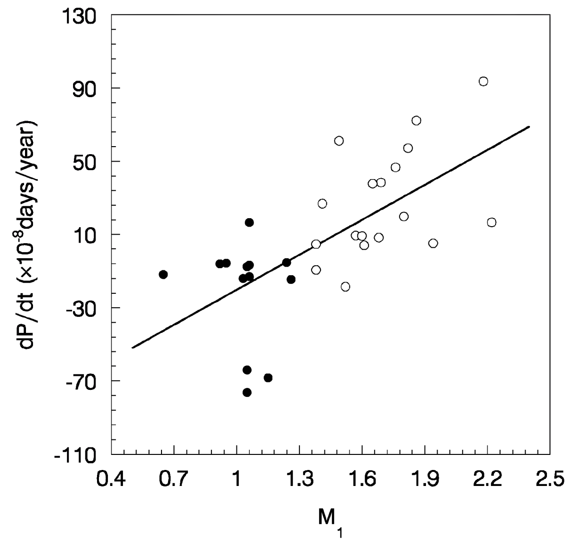

Figure 9. A possible relation between dP/dt and M_1 for low mass ratio ($q < 0.4$) overcontact binaries. Symbols are the same as in Fig. 7.

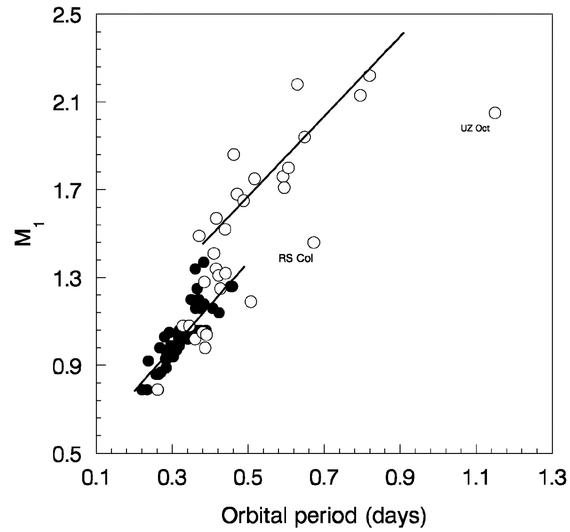

Figure 10. The relation between M_1 and P for observed overcontact binaries.

4 SOME STATISTICAL RELATIONS FOR OVERCONTACT BINARIES

A large self-consistent sample of absolute parameters for overcontact binaries (48 W type and 30 A type) were published by Maceroni & van't Veer (1996). After this study, spectroscopic elements of many W UMa-type binary stars were published by Rucinski and co-workers (Lu & Rucinski 1999; Rucinski & Lu 1999; Rucinski, Lu & Mochnacki 2000, 2001; Lu, Rucinski & Ogloza 2001). Using those elements the absolute parameters given by Maceroni & van't Veer (1996) are revised. However, for some systems there exists a large difference between the photometric mass ratio and the spectroscopic one. A detailed photometric investigation for those eclipsing binaries is planned. Using those parameters, the orbital angular momentum is calculated with the following equation:

$$J_{\text{orb}} = d^{1/2}\frac{M_1 M_2}{(M_1 + M_2)^{1/2}} \tag{18}$$

in units of $G^{1/2}\,R_{\odot}^{1/2}\,M_{\odot}^{3/2}$, where G is the gravitational constant and $M_{\odot}$ and $R_{\odot}$ are the mass and radius of the Sun, respectively. The specific angular momentum J_s is defined as $J_s = J_{\text{orb}}/(M_1 + M_2)$ in units of $G^{1/2}\,R_{\odot}^{1/2}\,M_{\odot}^{1/2}$. With these computed values, the M_1–P, J_s–M_1, J_s–M_2 and J_s–P diagrams can be formed.

4.1 M_1–P diagram

The M_1–P relation is presented in Fig. 10, where solid dots refer to W type and open circles to A type. As shown in this figure, this relation is quite tight and similar to the period–colour relation given by Eggen (1961, 1967) and by Rucinski (1985b). This may be caused by the fact that the temperature (colour index) is mainly determined by the mass of the primary. As displayed in Fig. 10, there exist two narrower sequences in this diagram: a first one starting at CC Com with the shortest period and mass (P = 0.221 and $M_1 = 0.79$), extending to large masses and long periods and a second sequence of large-mass systems, starting at a period of about 0.41 d and parallel to the first sequence. It is seen in this figure that systems on the second sequence are all A type with a larger mass, $M_1 >$ 1.35 $M_{\odot}$. Apart from RS Col and UZ Oct, the masses of systems on the first sequence are less than 1.35 $M_{\odot}$ and a transition between the two sequences takes place at $M_1 = 1.35\,M_{\odot}$ ($P = 0.41$ d), which may correspond to the p–p and CNO energy generation rate in the core.

For the first sequence, with least-squares method, the following relation is obtained:

$$M_1 = 0.391(\pm 0.059) + 1.96(\pm 0.17)P. \tag{19}$$

As for the second sequence, with the same method we can derive

$$M_1 = 0.761(\pm 0.150) + 1.82(\pm 0.28)P. \tag{20}$$

Mass is not an easily determinable stellar parameter, especially for overcontact binaries. It can only be obtained by knowing both photometric and spectroscopic elements. Overcontact binaries are usually dark stars rotating at an angular velocity 40–120 times faster than that of the Sun. This make it difficult to determine their spectroscopic parameters and only some were calculated. Since the orbital period is an easy parameter to determine, the M_1–P relation can be used to determine the parameters for overcontact binaries. Using equations (19) and (20), M_1 can be obtained. Then, combined with the photometric elements and using Kepler's third law, other parameters can be calculated.

Although the M_1–P relation is similar to the period–colour relation, the former has a more clearly physical meaning. Assuming conservative mass and angular momentum, the mass transfer from the primary to the secondary should cause both M_1 and P to decrease, and when mass transfer is from the secondary to the primary, both M_1 and P will increase. The strong correlation between M_1 and P may be the result of secular free mass transfer between both components during the evolution of the overcontact binary, which is assumed by TRO theory. However, if the mass transfer is conservative and if the M_1–P relation is caused by secular free mass transfer, we can expect that there should exist a strong relation between M_2 and P, which shows that the longer P is, the smaller M_2 will be. However, this kind of relation between M_2 and P does not exist. This may indicate that secular free mass transfer is not conservative. Mass and angular momentum loss cannot be avoided during the evolution. This will be discussed in following subsections.

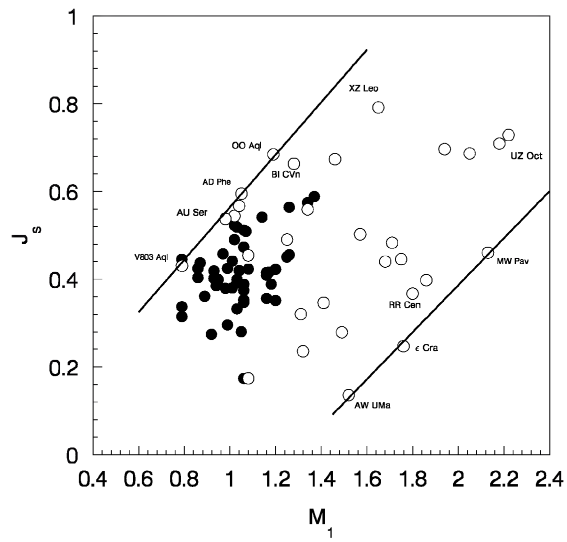

Figure 11. J_s–M_1 diagram for the observed overcontact binaries. Solid lines refer to the boundaries of the overcontact binary field.

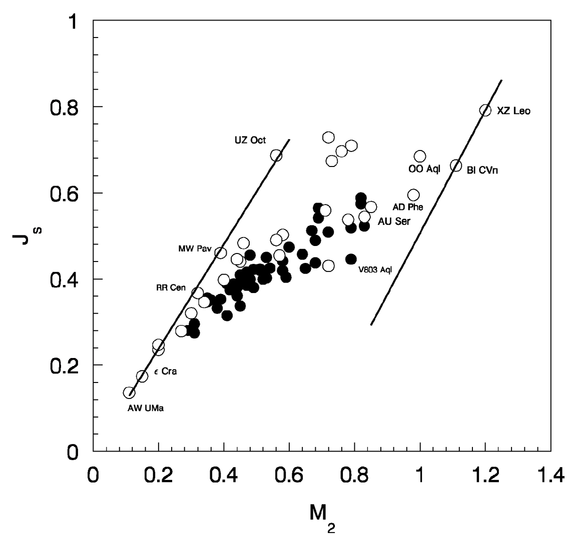

Figure 12. J_s–M_2 relation for the observed overcontact binaries.

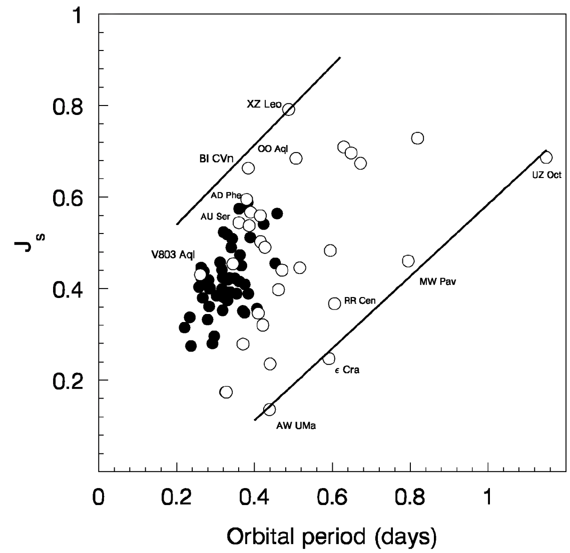

Figure 13. The J_s–P relation for overcontact binaries. Solid lines refer to the boundaries of the overcontact binary field.

4.2 J_s–M_1, J_s–M_2 and J_s–P diagrams

The relations between J_s and M_1, M_2 and P are plotted in Figs 11–13. As shown in Fig. 10, solid dots in these figures refer to W-type and open circles to A-type overcontact binary stars. As displayed in Figs 11–13, W UMa-type binaries have a stable field and the two solid lines represent the left- and the right-hand boundaries. The names of some systems on or near the boundaries (such as V803 Aql, AU Ser, OO Aql, BI CVn, XZ Leo, AW UMa, ϵ Cra and MW Pav) have been marked in these figures. When the distributions of the sample stars in the three figures are compared, the following interesting results can be obtained.

(i) Systems on or near the left-hand boundary in the J_s–M_1 diagram (e.g. V803 Aql, AU Ser, OO Aql, BI CVn and XZ Leo) are on or near the right-hand boundary of the J_s–M_2 one, and systems on or near the right-hand boundary of the J_s–M_1 diagram (e.g. AW UMa, ϵ Cra and MW Pav) are on or near the left-hand boundary in the J_s–M_2 diagram. These findings indicate that the distribution of overcontact binaries in the J_s–M_1 diagram is opposite to that in the J_s–M_2 diagram.

(ii) The distribution of W UMa-type binary stars in the J_s–M_1 diagram is similar to that in the J_s–P diagram. However, the distribution of overcontact binaries in the J_s–M_2 diagram is opposite to that in the J_s–P diagram.

What do these distributions mean? For an overcontact binary (e.g. V803 Aql, AU Ser, OO Aql, BI CVn and XZ Leo), when M_1 reaches its minimum value (near the left-hand boundary of the J_s–M_1 diagram), its M_2 value will be greatest (near the right-hand boundary of the J_s–M_2 diagram), at the same time, its orbital period reaches a minimum value (near the left-hand boundary of the J_s–P diagram). On the other hand, when M_1 of an overcontact binary (e.g. AW UMa, ϵ Cra and MW Pav) reaches its maximum value (on or near the right-hand boundary of the J_s–M_1 diagram), M_2 will be minimum (near the left-hand boundary of the J_s–M_2 diagram), while its orbital period reaches its largest value (on the right-hand boundary of the J_s–P diagram). As is known, if mass and angular momentum are conservative, mass transfer from the primary to the secondary would result in both M_1 and P decreasing, but M_2 increasing. In contrast, mass transfer from the secondary to the primary component can cause both P and M_1 to increase, but a decreasing M_2. The distributions of observed overcontact binaries in Figs 11–13 strongly suggests that there exists free mass transfer between the components, and secular mass transfers in both directions (i.e. from the primary to the secondary or from the secondary to the primary) are present in overcontact binaries.

Assuming conservation of mass and angular momentum, the evolution track of an overcontact binary will be parallel to the M_1 and the M_2 axes in Figs 11 and 12, respectively, and the lost mass ΔM from one component should be fully transferred to the other one. In this case, the width of the overcontact field in the J_s–M_1 diagram (ΔM_1) should be equal to that of the overcontact field in the J_s–M_2 diagram (ΔM_2). However, this is not the case. As shown in Figs 11 and 12, $\Delta M_1 \neq \Delta M_2$. This strongly suggests that the mass transfer between the two components is not conservative, overcontact

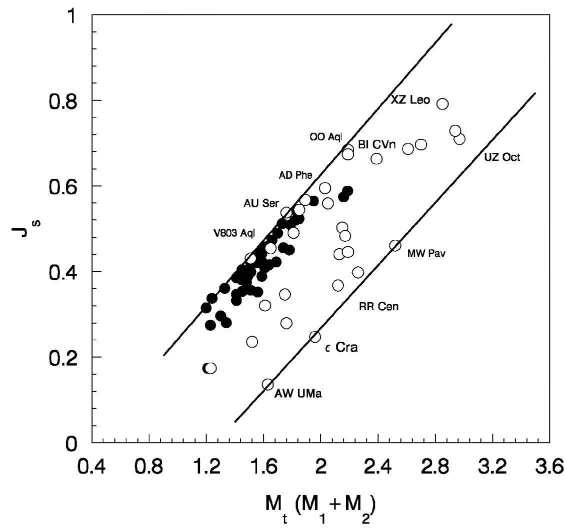

Figure 14. The same as in Figs 11 and 12, but for the total mass M_t.

binaries should lose mass during their evolution. W UMa-type stars are late-type systems with a rapid rotation (at an angular velocity 40–120 times faster than that of Sun). Thus these stars should have strong magnetic activity. The lost mass is coupling by the magnetic field, which can produce a large amount of angular momentum loss. In fact, the general trends of J_s–M_1, J_s–M_2 and J_s–P diagrams indicate that mass and angular momentum loss can play an important role in the secular evolution.

From the discussions in previous paragraphs, one may wonder whether those connections are made between systems which in reality have quite different total masses but similar M_1 or M_2 values. The correlation between J_s and the total mass M_t are plotted in Fig. 14 where the symbols are the same as in Figs 10–13. As shown in the figure, this relation is tighter than those for the J_s–M_1 and J_s–M_2 relations, and the distributions of observed overcontact binaries in the J_s–M_t diagram nearly resemble those in the J_s–M_1 diagram. This suggests that the total mass is mainly determined by the mass of the primary component M_1. Moreover, the comparisons between Figs 11–13 are, in fact, the comparisons between the values of M_1, M_2 and P for a given system. Thus the total mass has no influence to the connections between Figs 11–13.

5 DISCUSSIONS AND CONCLUSIONS

In the present paper, the orbital period changes of five W UMa-type stars (GW Cep, VY Cet, V700 Cyg, EM Lac, and AW Vir) are studied based on the analysis of their times of minimum light. Using the method proposed by Pringle (1975), the significance of the quadratic fit for each system is tested. The calculated λ values suggest that the quadratic fits for all systems are significant at more than the 99.99 per cent level. For VY Cet and V700 Cyg, a periodic oscillation is found superimposed on a long-term period increase. By considering the sinusoidal change to be caused by the presence of a third body, the orbital parameters of the third bodies for the two systems are determined in Subsections 2.3 and 2.4. On the other hand, since VY Cet and V700 Cyg both contain two late-type components, the cyclic period change could also be explained by magnetic activity cycles of the components as discussed by Applegate (1992).

Recent studies by Qian (2001a,b) have shown that the period change of a W UMa-type binary star is correlated with q and M_1. This conclusion is confirmed by the present statistical study of the period change. As displayed in Fig. 7, only cooler systems with a low mass ratio show a decreasing period. This may suggest that there may exist a critical mass ratio (q_{cr}) for the period change of an overcontact binary star, systems with $q > q_{cr}$ show an increasing period, while the periods of low mass ratio systems ($q < q_{cr}$) are decreasing. Moreover, the critical mass ratio is correlated with M_1. The larger M_1 is, the smaller q_{cr} will be. As seen in Fig. 7, when $M_1 > 1.6\ M_\odot$, no system shows a decreasing period. For the two systems CC Com and V523 Cas, with the shortest M_1 and P, the high-mass system (V523 Cas, $q = 0.57$) shows an increasing period, while the period of the low mass ratio one (CC Com, $q = 0.52$) is decreasing. The critical mass ratio is around $q_{cr} \sim 0.55$. When M_1 is increased to $M_1 = 1.35\ M_\odot$, q_{cr} is decreased to $q_{cr} \sim 0.072$.

As is shown in Fig. 10, all W-type stars are in the first sequence. Several A-type overcontact systems (e.g. VZ Psc, W Crv, RW Psa, AD Phe, AU Ser and OO Aql) with lower M_1 ($M_1 < 1.35\ M_\odot$) are also in this sequence. The one with the smallest M_1 and the shortest period is VZ Psc. However, the study by Hrivnak, Guiban & Lu (1995) showed that VZ Psc was in marginal contact with the secondary component being slightly detached. Thus strictly speaking, it is not an overcontact binary star. The recent study by Rucinski & Lu (2000) also indicated that W Crv is not an overcontact binary. For the other systems, they usually have a higher mass ratio ($q > 0.8$) and some show a sudden period decrease, e.g. AD Phe (Wolf et al. 2000), AU Ser (Qian, Liu & Yang 1999) and OO Aql (Demircan & Güral 1996). These systems may be at the beginning of the overcontact phase or have only recently evolved into overcontact. Thus, they are not included in the present statistical study of period change. The W UMa-type binary star was classified into two subgroups (A type and W type) by Binnendijk (1970), based on the properties of its light variation. Possibly, the W- and A-type classification is responsible for the mass of the primary component (or the spectral type and the temperature), i.e. for W type, $M_1 < 1.35\ M_\odot$ and for A type, $M_1 > 1.35\ M_\odot$, which corresponds to the p–p and CNO energy generation rate in the core. Therefore, the W-type phenomenon can be plausibly explained as being the result of the activity of star spots.

If we assume conservative mass transfer between the components, the properties of the period changes for W UMa-type stars may suggest that their evolution is controlled by q and this kind of system is oscillating around a critical mass ratio, and may not be oscillating between semidetached and overcontact configurations as predicted by TRO, because the period increase of high mass ratio systems ($q > q_{cr}$) corresponds to a mass transfer from the secondary to the primary. Meanwhile, the mass ratio of the system q will be decreasing, and when q is decreased to a small value ($q < q_{cr}$), the period decrease of the low mass ratio systems indicates that the direction of mass transfer is reversed, and thus q is increasing. As q exceeds the critical value, the period increases again and the system oscillates continuously. However, angular momentum is not conservative in practice, and it is not clear what mechanisms caused such an oscillation.

To avoid the braking stage of the TRO model, in 1981 Rahunen performed an interesting artificial computation to see what loss rate, $\mathrm{d}J/\mathrm{d}t$, is needed to keep a system in overcontact. His calculation showed that a critical AML rate ($\mathrm{d}\ln j/\mathrm{d}t \sim 2 \times 10^{-9}\ \mathrm{yr}^{-1}$) can keep W UMa stars in shallow overcontact. A larger AML rate will cause a system to soon reach the L_2-surface, and then to form a single star, while if a somewhat smaller loss rate is assumed, the system will be immediately broken and cyclic oscillation between overcontact and semidetached configurations occurs. Many observational data have suggested that W UMa stars showed strong magnetic activity. They are strong sources for soft X-ray and radio emissions and displayed star spots and chromospheric activity. The most likely mechanism for AML is magnetic braking via the coupling of the stellar wind with the magnetic field.

Recently, to explain the period change of W UMa systems, an evolutionary scenario was proposed by Qian (2001a,b). The scenario assumed that the variation of the degree of overcontact can cause the variation of magnetic activity thus changing the AML rate, which has been discussed by Vilhu (1981) and Smith (1984). The observed period decrease in a low mass ratio system means an increase of depth of overcontact, which causes a reduction in the AML rate. When the AML rate is lower than Rahunen's (1981) critical rate, the calculation by Rahunen (1981) has shown that the evolution of the system is mainly controlled by TRO, and thus the orbital period will be increasing and mass ratio decreasing. At the same time, the decreasing of mixing in the common envelope may result in a rather strong magnetic field, which causes a large AML rate (via the decreasing of the degree of overcontact). When the rate is larger than Rahunen's (1981) critical value, AML will control the evolution of the W UMa-type star and the orbital period will decrease again.

In the above evolutionary scheme, a W UMa-type binary cannot reach a semidetached configuration, since the period increase is accompanied by an increase in the AML rate, and as it is larger than Rahunen's critical value, the orbital period will decrease. On the other hand, the period decrease may cause a lower AML rate, and when the AML rate is less than the critical rate, the orbital period will increase again and the system cannot evolve into a single star soon. Thus the system may oscillate around a critical mass ratio. However, for the hotter overcontact binaries, since the rate of AML is lower, only when the system reaches an extremely low mass ratio ($q \sim 0.072$) is the period reversed and decreases. For many of these kinds of systems, overcontact may be broken and they will oscillate between semidetached and overcontact configurations as predicted by TRO. The dynamical evolution of this type of star have recently been discussed by Qian (2002).

The main problem with the evolutionary scheme is whether there is any observational evidence that has shown that the degree of overcontact couples with magnetic activity? Recently, *ROSAT* all-sky survey observations published by Stepien, Schmitt & Voges (2001) have shown that the X-ray flux of W UMa-type stars is about four to five times weaker than that of the fastest rotating single stars. This result may suggest that, for W UMa-type binaries, the levels of magnetic activities have been reduced by the existence of the common convective envelope (CCE). This conclusion can also be confirmed by the investigation of Hrivnak et al. (2001), who used ultraviolet data obtained with the *International Ultraviolet Explorer* (*IUE*) satellite, to study the chromospheric activity of the W UMa star, OO Aql. They pointed out that the short-period detached binary, ER Vul, has much higher levels of variable star spots and chromospheric activity than those of OO Aql. This also indicates that, when a late-type detached binary evolves into overcontact via magnetic braking, the previous high levels of activity may be significantly reduced by the formation of the CCE. All of these findings may suggest that, compared with the fast rotating single stars and short-period detached systems, the magnetic activity of W UMa-type stars has been reduced by the CCE. The thicker the CCE is, the weaker of magnetic activities will be.

In the previous subsections, the strong correlation between P and M_1, and the distributions of observed overcontact binaries in J_s–M_1, J_s–M_2 and J_s–P diagrams have been presented. These observational properties indicate that there exists a free matter exchange between the components and that overcontact binaries experience secular mass and angular momentum loss during their evolution. As shown in Figs 11–13, the observed overcontact binary stars have a stable field in these diagrams and the distributions of W UMa-type binary stars in these figures are strongly correlated. These findings suggest that the lost angular momentum can maintain a W UMa-type star in a shallow overcontact phase and it may oscillate around a critical mass ratio. These results are consistent with the predictions of the present evolutionary scenario.

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