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Late Evolution of Cataclysmic Variables

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ABSTRACT. We study the evolution of hydrogen-rich cataclysmic variables (CVs) near minimum orbital period at ∼78 minutes. As has been known for many years, these are among the most intrinsically common CVs, but they hide fairly well because of their faintness and low incidence of eruptions. We discuss their number and observational signatures, paying special attention to those that may have passed minimum orbital period—the "period bouncers." The status of binaries near minimum period is best determined by the mass ratio, and this is best constrained by measuring the accretion disk precession frequency, because that frequency is readily accessible to observation and proportional to the secondary star's mass. This method reveals four stars that are good candidates to have survived period bounce; two appear to have secondaries as puny as 0.02 $M_\odot$. But each star can have bounced only recently if at all. *There is still no strong evidence of any long era of evolution in a state of increasing period.* This conflicts sharply with discussions of observational data that have identified dozens of known CVs with this state. The total space density of cataclysmic variables is $\sim 10^{-5}$ pc^{-3}, with short-period systems constituting ∼75% of the total. Both estimates are far less than predicted by simple theories of evolution. It is probably necessary to have some means of destroying CVs before they reach the predicted very high space densities. This can be done by invoking an angular momentum loss mechanism that does not quickly subside as the mass ratio becomes very low.

1. INTRODUCTION

In the 1970s, the evolution of cataclysmic variable stars was still *terra incognita*. The most important breakthrough occurred when it was recognized that the main force driving the binary evolution is angular momentum loss, either by gravitational waves or by some kind of magnetic wind (Paczynski 1981; Paczynski & Sienkiewicz 1981; Verbunt & Zwaan 1981; Rappaport, Joss, & Webbink 1982, hereafter RJW). This finally provided a theoretical method to predict accretion rates. Study of those rates showed that angular momentum loss must rise sharply with orbital period (Patterson 1984, hereafter P84), and this has led to the popular view that long-period stars are driven mainly by a magnetic wind and short-period stars are driven mainly by gravitational radiation (GR). This simple dichotomy does not explain all features of the observational data but does explain the most obvious feature of the better studied stars: that accretion rates at short period are generally the lowest [$\sim(3\text{–}10) \times 10^{-11}$ $M_\odot$ yr^{-1}].

We still know little about the last phases of CV evolution, however. This is a serious gap, because hydrogen-rich binaries should evolve down to $P = 2$ hr in less than 10^9 yr, implying that most CVs should be at very short period. And indeed, observations *roughly* agree with this, suggesting ∼75% of CVs are below the 2–3 hr gap (Table 5 and Fig. 10 of P84). But the census in P84 suggested a serious shortfall. The evolution lifetimes of stars near the shortest periods (1.3 hr) should become drastically longer, as the secondary's thermal adjustment becomes unable to cope with the timescale for GR losses (Paczynski & Sienkiewicz 1981; RJW). This should cause a great pileup of old CVs at short period. Kolb (1993) studied this and predicted that 99% of CVs should be below the gap and 70% should have "bounced" off the minimum-period limit and now be evolving back toward longer period. Howell, Rappaport, & Politano (1997, hereafter HRP) gave similar numbers.

The present study was done to compare these figures with observation. Our estimates conflict somewhat with the predictions, and conflict sharply with HRP's description of the data. Even with selection allowances that we consider generous, we doubt that the fraction below the period gap exceeds 90%. This is a large discrepancy, at least a factor of 10. And we found only a moderate pileup at very short period. We identify four stars that seem to be good candidates for period bouncers, but there are not enough of them to dominate the CV population, unless one adopts extreme assumptions about observational selection.

2. ORBITAL PERIOD DISTRIBUTION

Evidence for period bounce could come from the distribution of orbital periods. Bounce produces more stars at the shortest orbital periods, because stars pass twice through that region, and *many* more if the evolution lifetimes lengthen considerably there, as most theories predict. In Figure 1 we present the

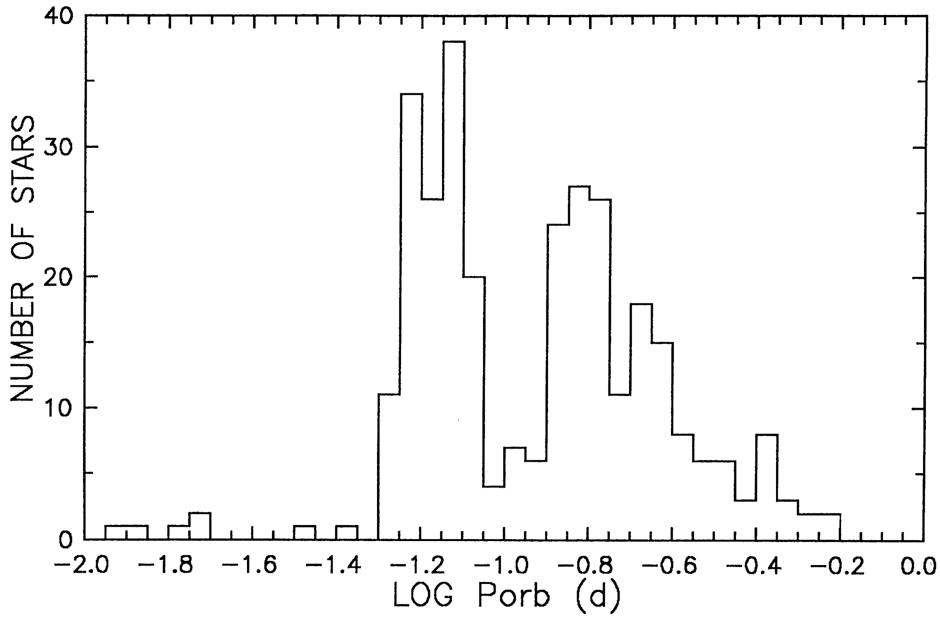

FIG. 1.—Number distribution of CVs with orbital period. Data are drawn from Ritter & Kolb (1998), updated to mid-1998.

observed distribution, complete as of mid-1998. The major features are as follows:

1. A few stars at very short period (<0.04 days), the helium-rich AM CVn binaries (Ulla 1994). Because the periods are so short and because many stellar properties are sensitive to *Z*/*A*, the AM CVn secondaries must be radically different from those in H-rich CVs. This guarantees a very different evolutionary path, and we do not consider these stars further in this paper.
2. The dearth of long-period stars, arising in part from rapid evolution through this phase.
3. The 2–3 hr period gap, arising perhaps from a detached state or rapid evolution through this phase. Theories abound for the physical origin of this (Rappaport, Verbunt, & Joss 1983; Spruit & Ritter 1987; Taam & Spruit 1989; Clemens et al. 1998).
4. The rise toward short periods, discussed earlier by P84.

Is the highest peak of Figure 1 consistent with a bounce? In other words, can most of these stars have reached this state through evolving past minimum period? No—at least not with a bounce near 1.3 hr. The reason is that those stars should subsequently evolve very slowly, remaining in the bins of shortest period and causing a huge pileup that is not observed. In § 4 we shall find another reason: that most of the secondaries appear to have masses appropriate for the main sequence.

3. ABSOLUTE MAGNITUDES

3.1. Distance Finding

Virtually all CVs of short period, and all candidate period bouncers, should be intrinsically quite faint. HRP and Sproats, Howell, & Mason (1996, hereafter SHM) identified several with quiescent magnitude $M_v = +13$–14. If true, this would be important. It was based on a single method of estimating distance, by assuming that the *K*-band surface brightness of the secondary star is constant and known. But for stars of very low mass, the surface brightness is neither known nor demonstrably constant (because the calibrators are of much higher mass; Bailey 1981; Ramseyer 1994). Further, the method gives only a lower limit to distance, unless the fractional contribution of the secondary to the *K* light is known. HRP and SHM recognized the latter in principle but underestimated its importance. In practice, one needs spectroscopy or ellipsoidal photometric variations to judge the contribution of the secondary; a single photometric color can establish the presence of a cool light source but cannot discriminate a secondary from accretion light (especially at low $\dot{M}$). And spectroscopy is largely inconsistent with the hypothesis that the secondaries dominate. Dhillon (1997) demonstrated that the *K*-band spectra of short-period CVs generally show emission lines and smooth continua, not the absorptions characteristic of cool secondaries.

There is no single technique, no "magic bullet," for distance finding to cataclysmic variables. But CVs are rich in variety and leave a rich variety of clues, discussed at length by P84 and Warner (1987, 1995a). Since the results were in substantial agreement, we shall not cover those reviews here. We have prepared a new version of Table 1 of P84, with improved calibrations, giving distance estimates for about 50% more stars. The most important improvement concerns dwarf novae in outburst, which we now know to be fairly good standard candles. The reason is that accretion disk areas are about equal (scaling weakly with period), and the thermal instability that

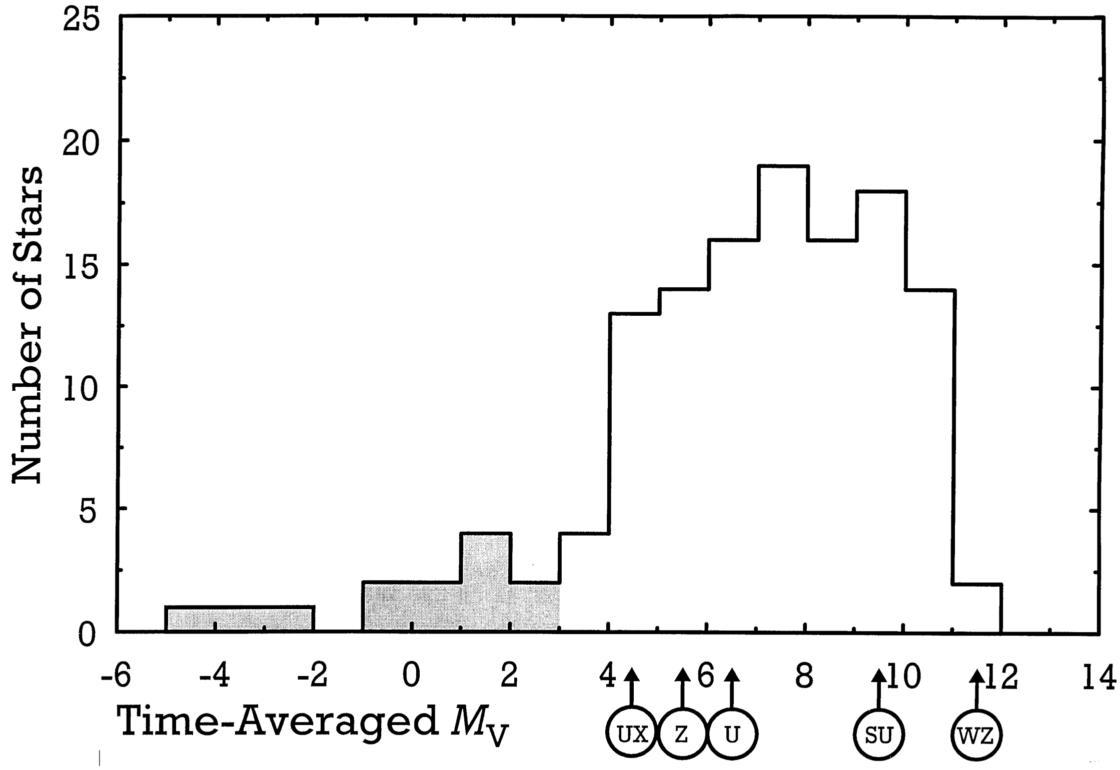

FIG. 2.—Distribution with time-averaged absolute visual magnitude. Stars in the shaded region are supersoft binaries. Accretion-powered stars range from about +3 to +12. The arrows at bottom indicate mean values for various classes (UX UMa, Z Cam, U Gem, SU UMa, WZ Sge).

causes the outburst shuts off when the disk reaches its hot state, i.e., when T_{eff} rises to ~20,000 K. For this high a temperature, M_v is relatively insensitive to T_{eff}, and we know from *UBV* photometry that the colors of dwarf novae in outburst are very uniform ($B - V = 0$, $U - B = -0.8$). The near constancy of T_{eff} and the emitting area leads to a near-constant M_v. This has been explained clearly and elegantly (Warner 1987; Cannizzo 1998). Warner's inclination-corrected empirical correlation is worth repeating:[1]

$$(M_v)_{max} = 5.74 - 0.259P\,[\mathrm{hr}]\,. \tag{1}$$

3.2. Time-averaged M_v

With these methods, we estimated distances and used eruption histories to calculate time-averaged values of M_v. Full details will be presented elsewhere, but one result is of interest here. We studied the distribution of CVs with $\langle M_v \rangle$ to see, for example, whether there might be some "second family" at very faint $\langle M_v \rangle$. Figure 2 shows the result. As in P84, we restricted ourselves to stars of known orbital period, which improves data reliability and is the key to constraining the secondary. The main distribution ranges from +3 to +11.6, with a long tail of very bright stars. The latter are the well-known supersoft binaries, which reach much greater luminosity by having access to a more powerful energy source, nuclear burning of accreted hydrogen (van den Heuvel et al. 1992).

At the faint end, we found no second family and no long tail. On the contrary, we found a fairly abrupt edge with not a single star fainter than $\langle M_v \rangle = +11.6$. This conflicts with HRP, who reported four stars with a quiescent $M_v = 13$–14 and stated errors of ± 0.5 mag. Three met our "known *P*" criterion, yielding a quiescent $M_v = +8.2$ (AH Eri), +9.5 (AY Lyr), and +12.2 (AL Com), with errors of ± 1 mag and a time-averaged M_v about 1–2 mag brighter. Such a tail is plausible on theoretical grounds and might still *exist*, but the stated evidence is spurious.[2]

3.3. Mass Transfer Cycles: A Skeleton in the Closet

The value of $\langle M_v \rangle$ is a useful probe of evolutionary status but is not entirely reliable. There are always subtleties in dis-

[1] Of course, this is no magic bullet either! But it is a very useful clue for most dwarf novae, which tend to erupt to a fairly repeatable brightness level. Extra care should be taken when applying it to superoutbursts or any star with only a sparsely known eruption history.

[2] The main reason was the overzealous and exclusive use of the *K*-band surface brightness relation, as discussed above. In the most striking cases, it led to assignments of M_v around +8 to +9 in outburst, which makes for great difficulty—it gives a dwarf nova "no reason to erupt."

tance finding, although these are ameliorated in such faint stars because the relatively tractable white dwarf is often prominent in the flux distribution. More serious is the issue of long-term cycles of mass transfer, stimulated by irradiation of the secondary (Wu, Wickramasinghe, Warner 1995; King 1995; King et al. 1996; McCormick & Frank 1998). If such cycles exist, then our deductions of $\langle M_v \rangle$ are merely snapshots; averages over a class of stars would still be reliable, but not so much the results for any individual star.

4. SECONDARY STAR MASSES

4.1. What's Normal, Anyway?

The most telling signature of period bounce is a low mass for the secondary. The reason is simply that the secondary continues to lose mass inexorably in evolution, so the farther it has bounced, the farther the secondary departs from "normal" secondaries in binaries of decreasing period. Detection of abnormally low masses in secondaries would be impressive evidence for period bounce.

But the notion of normality here needs clarification. Kepler's third law in Roche geometry constrains the secondary to obey the period-density relation

$$P\,[\mathrm{hr}] = 8.75 \left(\frac{M}{R^3}\right)^{-1/2}, \quad (2)$$

with M and R in solar units (Faulkner, Flannery, & Warner 1972). Thus, all short-period CVs must have secondaries that are dense (160 $\rho_\odot$ at 1.4 hr). Most are likely near the main sequence, which can be expressed by

$$\frac{R_2}{R_\odot} = \alpha \left(\frac{M_2}{M_\odot}\right)^{\beta}, \quad (3)$$

where, for example, $\alpha = 1$ and $\beta = 0.88$ (the choice of P84). Recent improvements in parallaxes, color-T_e calibrations, and theoretical models of low-mass stars suggest slightly lower values of α and β. A recent and thorough discussion is given by Clemens et al. (1998, hereafter C98). Although the results have not greatly changed, we can now have much greater confidence in applying these relations to CVs, since they are now based on stars of comparable mass, all the way to 0.08 $M_\odot$. The data in C98's Figure 3 suggest an overall relation with $\alpha = 0.90$, $\beta = 0.80$, but with perhaps a few bumps. Because we are concerned here with the lowest masses, we restrict ourselves to the range 0.08–0.18 $M_\odot$ and approximate their data with

$$\left(\frac{R}{R_\odot}\right) = 0.81 \left(\frac{M}{M_\odot}\right)^{0.74}. \quad (4)$$

We shall use this relation to define "normal," since it is empirically based. However, these radii are still 10%–25%

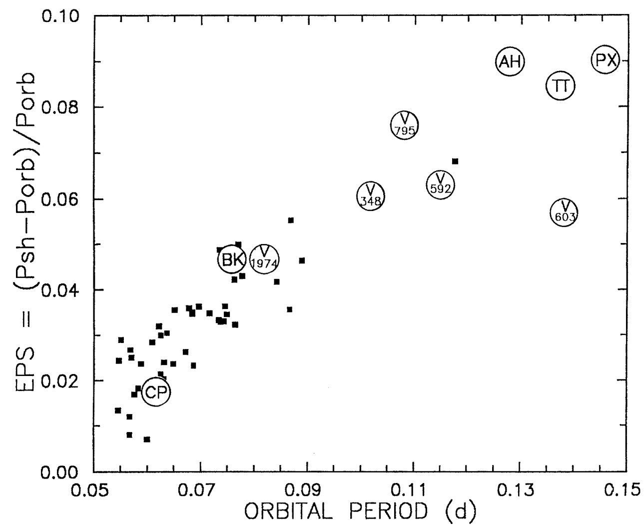

FIG. 3.—The ϵ-P relation for positive superhumpers. SU UMa stars are shown by squares, while permanent superhumpers are labeled by name. Generally ϵ scales with P, but there are a few low points at very short period. Error bars are given in Table 1 and are comparable to the symbol size.

larger than predicted by theoretical mass-radius relations in this mass range (Baraffe & Chabrier 1996; Tout et al. 1996; D'Antona & Mazzitelli 1994); and as emphasized by C98, we still do not know with assurance whether the flaw lies in the models or the observations. So that is a reason for caution.

Equations (2) and (4) then imply

$$\frac{M_2}{M_\odot} = 0.048 \left(\frac{P}{1\ \mathrm{hr}}\right)^{1.64}. \quad (5)$$

Thus a secondary of 0.04 $M_\odot$ at $P = 1.4$ hr is undermassive by a factor of 2 ("abnormal"). This does have the interesting and arguably happy feature that the end of the main sequence at 0.08 $M_\odot$ is encountered at the end of the period distribution at 1.3 hr. But the main point here is that postbounce systems should have secondaries of mass lower than given in equation (5).

4.2. Masses from Radial Velocities

Radial velocity studies are the traditional way to learn stellar masses. At $P = 1.5$ hr, a normal CV would have $M_1 = 0.7\ M_\odot$, $M_2 = 0.10\ M_\odot$; a CV bouncing at 1.3 hr would have its secondary whittled down to ~0.03 $M_\odot$ by the time $P = 1.5$ hr is reached after bounce. The solution of Kepler's laws for an edge-on system yields $K_1 = 58$, $K_2 = 436$ km s^{-1} prebounce; and $K_1 = 20$, $K_2 = 480$ km s^{-1} postbounce. (In this rough discussion, we assume for convenience and pleasantness that all binaries are edge-on, with white dwarfs of 0.7 $M_\odot$.) Thus measurement of K_1 would seem to be a good discriminant.

But on closer inspection this method fails, for at least two reasons. First, the practical limit for the measurement of broad lines in faint stars is ~30 km s^{-1}, not sufficiently sensitive to reveal secondaries of very low mass. And second, spectroscopic measurement of K_1 is always confounded by the presence of a contaminating wave of larger amplitude ("s-wave"), probably associated with the mass-transfer stream that wheels around once per orbit and is rich in emission lines. In every CV where eclipses reveal the precise moment of the stars' dynamical conjunction, we always find v_{rad} curves shifted from their expected phases, by amounts up to 90°. This implies that the measured K_1 cannot be trusted as a tracer of the true motion. The problem is worsened if the true motion is very small, because the mass-transfer stream has a velocity of at least 300 km s^{-1}, and any signal at $K_1 = 20$ will be swamped.

4.3. Masses from Superhumps

Although traditional measurement of M_2 via K_1 seems unattainable, there is a promising indirect method. This exploits the fact that most short-period CVs are dwarf novae of the SU UMa type, which show photometric "superhumps" during their long eruptions. Superhumps occur at a period slightly longer than the binary orbit and probably arise from accretion disk precession, forced by gravitational perturbation from the secondary after the disk is rendered eccentric by the (largely unrelated) instability that occurs at the 3:1 disk resonance. The displacement of the superhump period P_{sh} from the orbital period P reveals the precession rate via

$$(P_{prec})^{-1} = (P)^{-1} - (P_{sh})^{-1}. \qquad (6)$$

P_{sh} is easy to measure in superoutburst, and P is usually obtainable from spectroscopy or photometry in quiescence.

A simple and useful fact is that the precession rate should be proportional to the perturbing mass (the secondary). In particular, the classical precession rate of a single particle at a fixed radius R in the disk is given by the nonresonant formula

$$\frac{\Omega_{prec}}{\Omega} = \frac{3}{4}\frac{q}{(q+1)^{1/2}}\left(\frac{R}{a}\right)^{3/2}, \qquad (7)$$

where a is the binary separation, Ω is the orbital frequency, and $q = M_2/M_1$ (Osaki 1985). At the 3:1 resonance (the most likely site for the excitation of the eccentricity; Whitehurst 1988; Osaki 1989; Lubow 1991), we have approximately $R = 0.46a$ and hence

$$\frac{\Omega_{prec}}{\Omega} = \frac{0.23q}{(q+1)^{1/2}}. \qquad (8)$$

So the precession and orbital frequencies, each easily measured, determine the mass ratio through equation (8). Mineshige, Hirose, & Osaki (1992) and Warner (1995b) discuss other applications of this relation.

Table 1 contains the data for the 53 stars with accurately known P and P_{sh} as of mid-1998. It includes only stars with "common" or "positive" superhumps ($P_{sh} > P$). Forty-two have all the defining credentials of SU UMa–type dwarf novae. The remaining 11 are more or less permanently found in a high accretion state ("permanent superhumpers") but are included here since they show no obvious differences in precession. Three ultrashort-period stars (AM CVn, CR Boo, and V485 Cen) are also included but not further studied, because they are helium-rich.

Accuracy in the periods is critical for the use of Table 1. We omit stars where the uncertainty in either period exceeds 0.4%. For dwarf novae showing small changes in P_{sh}, we standardized by adopting a period equal to that achieved 4 days after superhump onset; this was quite close to the mean period. References cited are mainly the sources of the accurate periods, and we usually adopt the published estimate of uncertainty. Where no estimate or an unreasonably small estimate was given, we supply our own estimate. (In practice, periods can be measured to a typical 1 σ accuracy of ~0.06 cycles divided by the time baseline.) For permanent superhumpers, we adopt an uncertainty that spans the range of observed periods, assuming observation baselines greater than 10 days.

To deal with strict observables, we define the fractional period excess $\epsilon = (P_{sh} - P)/P$. Stolz & Schoembs (1984) first noticed that ϵ grows with P, and Figure 3 shows the empirical correlation. For $P > 0.06$ days, ϵ increases smoothly with P; but for the eight stars of shortest period, that correlation certainly breaks down. Four short-period dwarf novae have values of ϵ about a factor of 2–3 below the main cluster.

Equations (6) and (8) determine $\epsilon(q)$, roughly

$$\epsilon = \frac{0.23q}{1 + 0.27q}, \qquad (9)$$

and q is a function of period for the simplest possible model of a CV (ZAMS secondary, M_1 fixed at 0.7 $M_\odot$). With equation (5) we obtain an ϵ-P relation given by the bold curve in Figure 4. Here we have changed to a logarithmic scale to study the behavior at short period, and drop explicit symbols for the permanent superhumpers since they appear to follow the same trend. In this simple theory, ϵ depends on q alone, and at the right of Figure 4 we supply a scale with the appropriate values of q (and M_2, assuming $M_1 = 0.7\ M_\odot$ in all cases). The four stars with very low ϵ are shown by name and have an estimated $M_2 \sim 0.02$–$0.04\ M_\odot$.

The shaded region around the theoretical curve indicates the effect of propagating a 10% error in R and a 20% error in M_1, added in quadrature. We chose a standard $M_1 = 0.7\ M_\odot$ since it is close to the advertised mean value for single white dwarfs (0.6 $M_\odot$; Weidemann 1990), is close to the advertised mean value for white dwarfs in CVs (0.74 $M_\odot$; Webbink 1990), and is about the geometric mean between the lightest and heaviest white dwarfs ever measured. To summarize: we measure

TABLE 1
PERIODS OF POSITIVE SUPERHUMPERS

Star	P_{orb} (days)	P_{sh} (days)	ϵ	Source	Star	P_{orb} (days)	P_{sh} (days)	ϵ	Source
AM CVn*	0.011906(1)	0.012166(1)	0.0218(1)	1	CY UMa	0.06957(4)	0.07210(9)	0.0364(14)	28
CR Boo	0.017029(2)	0.01723(2)	0.0117(12)	2	FO And	0.07161(18)	0.0741(2)	0.0349(40)	29
V485 Cen	0.040995(1)	0.04216(2)	0.0284(5)	3	VZ Pyx	0.07332(3)	0.07576(15)	0.0333(20)	30
DI UMa	0.054564(2)	0.05529(3)	0.0133(5)	4	CC Cnc	0.07352(5)	0.0771(2)	0.0487(27)	31
V844 Her	0.054643(7)	0.05597(5)	0.0243(9)	5	HT Cas	0.07364721(1)	0.0761(2)	0.0330(30)	32
LL And	0.055053(5)	0.0567(2)	0.0296(36)	6	VW Hyi	0.07427104(1)	0.07673(6)	0.0331(8)	33
AL Com	0.0566684(1)	0.05735(3)	0.0120(7)	7	Z Cha	0.07449926(1)	0.07721(7)	0.0364(9)	34
WZ Sge	0.05668785(1)	0.05714(4)	0.0080(6)	8	WX Hyi	0.0748134(2)	0.0774(1)	0.0346(14)	35
SW UMa	0.05681(14)	0.05820(6)	0.0245(27)	9	BK Lyn*	0.07498(5)	0.07857(1)	0.0479(7)	36
HV Vir	0.05711(6)	0.05833(5)	0.0214(11)	10	AW Gem	0.07621(10)	0.07943(15)	0.0422(27)	37
MM Hya	0.057590(1)	0.05865(6)	0.0184(10)	11	SU UMa	0.07635(5)	0.07877(6)	0.0317(12)	38
WX Cet	0.05829(4)	0.05936(3)	0.0183(8)	12	HS Vir	0.07696(17)	0.0808(1)	0.0499(23)	39
EG Cnc	0.05997(9)	0.06037(3)	0.0067(8)	13	V503 Cyg	0.0777(2)	0.08104(7)	0.0430(27)	40
T Leo	0.058819(7)	0.06021(8)	0.0236(14)	14	V1974 Cyg*	0.08126(1)	0.08509(8)	0.0471(10)	41
AQ Eri	0.06094(6)	0.06267(13)	0.0284(21)	15	TY PsA	0.08414(18)	0.08765(12)	0.0417(22)	42
CP Pup*	0.06145(6)	0.0625(1)	0.0171(20)	16	DV UMa	0.0858527(1)	0.08875(20)	0.0337(24)	43
V1159 Ori	0.06218(1)	0.06417(7)	0.0320(11)	17	YZ Cnc	0.0868(2)	0.09160(15)	0.0553(26)	44
V436 Cen	0.0625(2)	0.06383(8)	0.0212(32)	18	GX Cas	0.0889(3)	0.09302(7)	0.0463(36)	45
HO Del	0.0625(3)	0.0644(2)	0.0304(50)	19	V348 Pup*	0.101839(1)	0.1084(4)	0.0640(40)	46
VY Aqr	0.06309(4)	0.06437(9)	0.0203(15)	20	V795 Her*	0.108265(1)	0.1165(1)	0.0760(10)	47
OY Car	0.0631209(1)	0.06440(10)	0.0203(15)	21	V592 Cas*	0.115063(1)	0.12226(6)	0.0625(5)	48
ER UMa	0.06366(3)	0.06566(8)	0.0314(11)	22	TU Men	0.1172(2)	0.1256(2)	0.0717(32)	49
UV Per	0.06489(11)	0.06641(7)	0.0234(23)	23	AH Men*	0.12721(6)	0.1385(2)	0.0887(16)	50
SX LMi	0.06717(11)	0.06950(7)	0.0347(25)	24	TT Ari*	0.1375511(2)	0.1492(1)	0.0847(7)	51
SS UMi	0.06778(4)	0.07022(10)	0.0360(15)	25	V603 Aql*	0.1381(1)	0.1460(7)	0.0572(51)	52
TY Psc	0.06833(5)	0.0707(1)	0.0347(15)	26	PX And*	0.146353(1)	0.1595(2)	0.0898(14)	53
IR Gem	0.0684(3)	0.0708(3)	0.0351(66)	27					

NOTES.—Superhump periods are slightly unstable. For permanent superhumpers, we adopted the mean of observed values, with the error giving the range. For dwarf novae, we adopted the mean value or, in the case of sparse data, an estimate 4 days after superhump onset. This reproduced the mean value for stars where both quantities could be observed. The estimated error in P_{sh} for dwarf novae is the error in estimating this quantity, not the full range of variation in P_{sh} (which is considerably larger). The asterisks (*) denote stars other than dwarf novae.

REFERENCES.—(1) Harvey et al. 1998; Provencal et al. 1995. (2) Provencal et al. 1997; Wood et al. 1987. (3) Augusteijn et al. 1996; Olech 1997. (4) Kato et al. 1996a; Fried et al. 1998. (5) Vanmunster et al. 1998. (6) T. Kato 1998; private communication; Kemp et al. 1998. (7) Kato et al. 1996b; Patterson et al. 1996. (8) Patterson et al. 1981. (9) Robinson et al. 1987; Shafter, Szkody, & Thorstensen 1986; Semeniuk et al. 1997. (10) Leibowitz et al. 1994; Kemp & Patterson 1998. (11) Misselt & Shafter 1995; Kemp et al. 1998. (12) O'Donoghue et al. 1991; Thorstensen et al. 1996. (13) Patterson et al. 1998a. (14) Shafter & Szkody 1984; Lemm et al. 1993. (15) Thorstensen et al. 1996; Kato 1991. (16) Patterson & Warner 1998; O'Donoghue et al. 1989. (17) Thorstensen et al. 1997; Patterson et al. 1995. (18) Gilliland 1982; Warner 1983; Semeniuk 1980. (19) Vanmunster et al. 1998. (20) Thorstensen & Taylor 1997; Patterson et al. 1993. (21) Wood et al. 1989; Hessman et al. 1992. (22) Thorstensen et al. 1997; Kato & Kunjaya 1995; Misselt & Shafter 1995. (23) Thorstensen & Taylor 1997; Udalski and Pych 1992. (24) Wagner et al. 1998; Nogami, Masuda, & Kato 1997. (25) Thorstensen et al. 1996; Chen, Liu, & Wei 1991; Vanmunster 1998. (26) Thorstensen et al. 1996; Vanmunster et al. 1998. (27) Szkody, Shafter, & Cowley 1984. (28) Thorstensen et al. 1996; Harvey & Patterson 1995. (29) Thorstensen et al. 1996; Kato 1995. (30) Thorstensen 1997; Remillard et al. 1994; Kato & Nogami 1997b. (31) Thorstensen 1997; Kato & Nogami 1997a. (32) Zhang, Robinson, & Nather 1986. (33) Van Amerongen et al. 1987; Vogt 1983. (34) Warner & O'Donoghue 1988; Robinson et al. 1995. (35) Schoembs & Vogt 1981; Bailey 1979b. (36) Skillman & Patterson 1993; Ringwald et al. 1996. (37) Kato 1996. (38) Thorstensen, Wade, & Oke 1986; Udalski 1990. (39) Vanmunster et al. 1998. (40) Harvey et al. 1995. (41) Retter et al. 1997; Skillman et al. 1997. (42) Warner, O'Donoghue, & Wargau 1989; O'Donoghue & Soltynski 1992. (43) Skillman et al. 1998b. (44) Shafter & Hessman 1988; Patterson 1979. (45) Nogami et al. 1998; Skillman et al. 1998b. (46) Baptista et al. 1996; Tuohy et al. 1990; Patterson 1998. (47) Shafter et al. 1990; Zhang et al. 1991. (48) Taylor et al. 1998. (49) Stolz & Schoembs 1984; Mennickent 1995. (50) Patterson 1995; Patterson et al. 1998b. (51) Thorstensen, Smak, & Hessman 1985; Skillman et al. 1998a. (52) Patterson et al. 1997. (53) Thorstensen et al. 1991; Patterson et al. 1998b.

ϵ, deduce q with equation (9), estimate M_2 as accurately as we guess M_1, and compare with theory by adopting a specific mass-radius relation, viz., that of main-sequence stars.

It is noteworthy that the probable mass ratios range up to ~0.5, whereas theoretical calculations indicate that the eccentric instability producing the superhumps is limited to $q \lesssim 0.25$. Theorists should work harder on this point.

4.4. Checking $\epsilon(q)$

Since we rely on ϵ to learn q, it is important to know if we find the correct answer for stars of known q. The only stars with q known from "classical" binary-star methods (Roche geometry and Kepler's laws) are the four well-studied eclipsers in Table 1. Figure 5 shows the $\epsilon(q)$ correlation, with equation

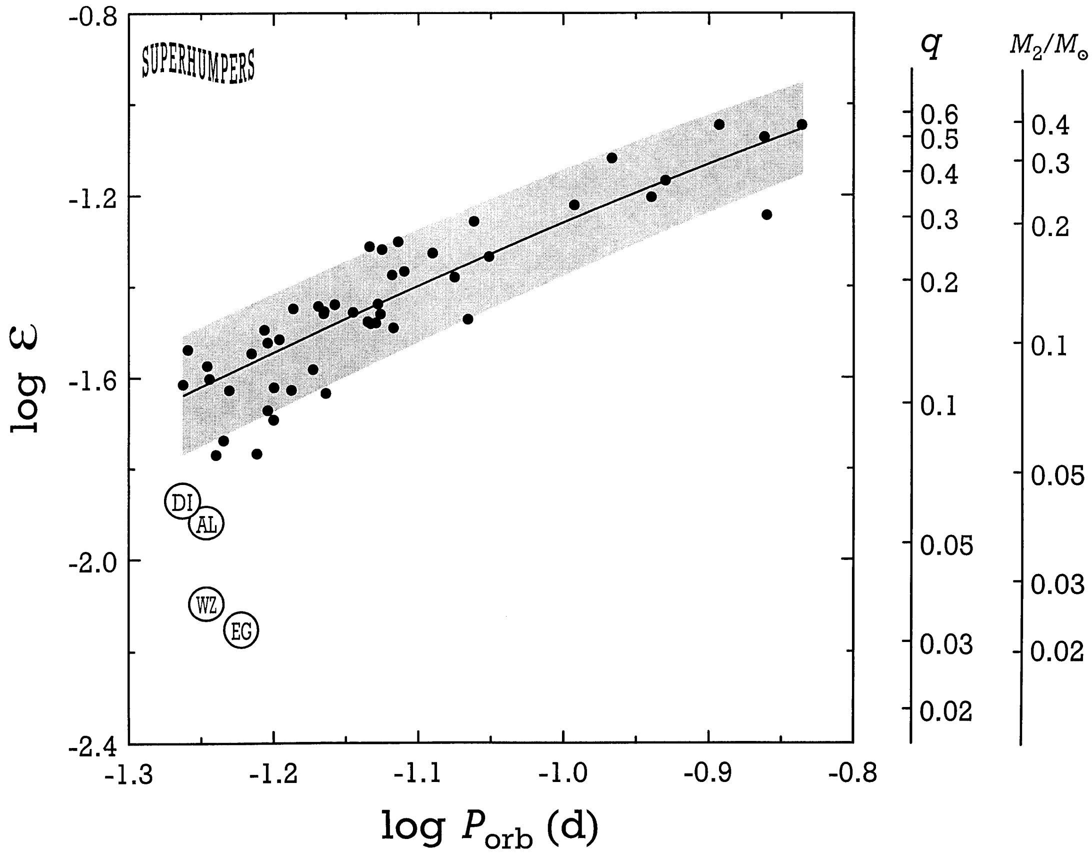

FIG. 4.—The ϵ-P relation in log-log space. The stars of very low ϵ are labeled by name. Deduced values of q and M_2, according to eq. (9), are shown at right. The curve indicates the theoretical $\epsilon(P)$ relation, coupling eq. (9) with the main-sequence assumption of eq. (5). Shading indicates the range accessible with a 10% uncertainty in the secondary's radius, and a 20% uncertainty in M_1, added in quadrature.

(9) superimposed. Within the limits of such sparse data, equation (9) survives the comparison.

4.5. Apologia Pro Relationis Superhumpis

The really big surprise is that this theory does not fail miserably! Consider how unpromising are its origins. We relied on a resonant process to make the superhumps. We then represented the disk as consisting of noninteracting particles, despite the vastly greater complexity in a real disk, and (inconsistently) adopted a precession formula appropriate for nonresonant orbits. And yet there it is, Figure 4, a fairly good fit with no adjusted parameters.

It may well turn out that this agreement is accidental—for example, that we have neglected two large effects that fortuitously cancel. This is no mere hiccup of conservatism. In fact, it is even likely, because there are more precise calculations (Hirose & Osaki 1990, 1993; Whitehurst & King 1991; Molnar & Kobulnicky 1992; Lubow 1992) that suggest precession frequencies up to a factor of 2 discrepant from equation (8). (But those calculations are also discrepant from each other by up to a factor of 2, which is why we have elected to use a simple algebraic formula.) The important point really concerns the *slope* of the curve in Figure 4. The observed slope for $P >$ 0.06 days is what main-sequence secondaries produce, and this is not likely to be an accident. The main part of the interpretation peddled here depends only on the most elementary aspect of the theory: that the perturbation scales with M_2.

The fairly low dispersion in ϵ at a given P(>0.06 days) is also noteworthy. It suggests a low dispersion in white dwarf masses. Or, more conservatively, that any flaw in this assumption is being masked by a systematic cancelling flaw in the secondary's assumed mass-radius relation (in the sense of greater M_1 associated with greater departure from main-sequence structure). The location of period minimum also depends somewhat on M_1($P_{\rm min} \propto M_1^{0.23}$); and since it appears to be pretty sharp at 1.3 hr, we suspect that the simpler explanation—low dispersion in M_1—is correct.

4.6. The Lowest Points

The four lowest points in Figure 4 signify mass ratios near 0.05, about a factor of 2–3 below the points seeming to define the main trend with P. This could result from a very high

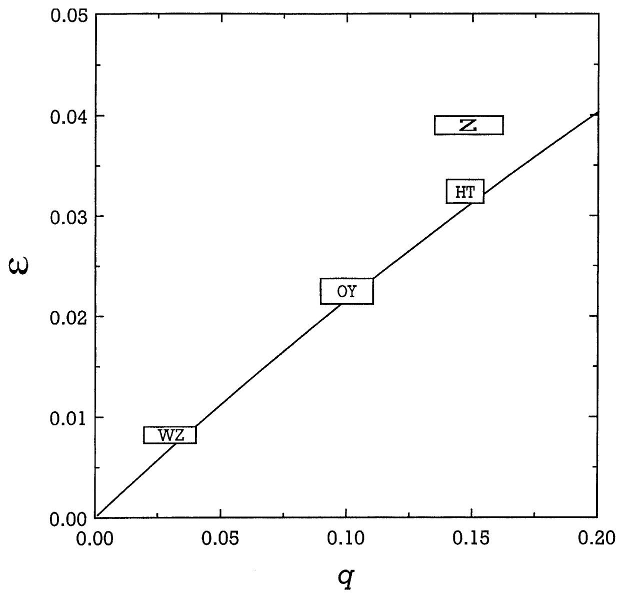

FIG. 5.—A comparison between eq. (9) and measured values of ϵ and q in the four eclipsing stars with known q. Sources of q are Wood, Horne, & Vennes (1992; OY Car), Horne, Wood, & Stiening (1991; HT Cas), Wade & Horne (1988; Z Cha), and this paper and Ritter & Schroeder (1979; WZ Sge).

M_1($\sim$ 1.4 $M_\odot$), but that is very unlikely since it would lead to expectations that are not fulfilled (higher accretion luminosity, strong X-ray emission, classical nova eruptions, very high Stark broadening of absorption lines). On the contrary, three of these stars are notable for their intrinsic faintness ($M_v \sim +12$) and low accretion rate. Thus it is more probable that they reach low q by having low M_2. The *prima facie* evidence suggests $M_2 = 0.02$–0.05 $M_\odot$, and 0.06–0.10 $M_\odot$ for the cluster of stars with higher ϵ but similar P.

Could these M_2 estimates be systematically distorted by the admittedly unreasonable simplicity of the model? Yes, certainly. But it must then be reckoned surprising that the model manages to reproduce the correct $\epsilon(P)$ dependence at longer P. Perhaps a better way to state this result is the following. A high dispersion in ϵ occurs only at short period. This is unlikely to arise from a high dispersion in M_1, on the grounds just mentioned, and also because any plausible change in M_1 as a result of evolution (e.g., due to classical nova eruptions) should have occurred earlier. It is more likely to come from dispersion in M_2. And our knowledge of evolution still only equips us to understand how H-rich stars can become larger than their main-sequence radius, not smaller. Then the secondaries in those four stars should be considered to have larger radii—or, equivalently, be “undermassive.”

As the secondaries are whittled down in mass, their thermal (Kelvin-Helmholtz) timescales become long (because $t_{\rm KH} \propto M^{-3}$), yet angular momentum loss throttles the star and forces it to lose mass on another timescale that is relatively short. Therefore, the secondaries become progressively more unable to contract to keep pace with their diminished mass. Roughly speaking, they have too much thermal energy to shrink, so remain at about constant radius.[3] Calculation shows that a small expansion may also occur, as the star’s mass-radius relation changes due to the onset of degeneracy. By following that expansion, we can track the consequent change in q and ϵ.

We have done this using Figure 2 of HRP, and we show the result as the curve in our Figure 6. The numbers along the curve indicate the model ages in Gyr. The curve shows a “period bounce” at ~1.1 hr and tracks back to the lower right, passing fairly near the lowest points.

Is this evidence for period bounce? Well, somewhat. At least it looks like the type of evidence that could, with more work, sensitively test the hypothesis. But the data do not yet define the curve well enough at short P to establish period bounce. For example, if the period minimum is considered “soft” (depending on M_1, or on the secondary’s helium content, which drastically affects $P_{\rm min}$ through affecting mean molecular weight) in the range 1.3–1.5 hr, then an alternative interpretation is that stars plummet more or less vertically at the lower left of Figure 4. That would imply that stars never really bounce in period, but merely evaporate their secondaries near minimum period. In either case, the evidence seems good that there are a few secondaries of very low mass, and it is plausible that they reach that state through the loss of thermal equilibrium setting in around 0.08 $M_\odot$.

4.7. And the Points That Aren’t There

Perhaps the most surprising feature of Figure 4 is the absence of any stars with $P > 0.06$ days and ϵ well below the main cluster. The simplest interpretation is that it signifies *the rarity of active CVs that have evolved far beyond period minimum.*

Can this arise from the finite age of the Galaxy? Probably not. The HRP model reaches the vicinity of EG Cnc, the “oldest living CV,” at 6 Gyr. This is likely an overestimate, because that model reaches minimum period at 1.1 hr, not 1.3 hr as observed. We do not yet understand the reason for the discrepancy (possibly that angular momentum loss exceeds that due to gravitational radiation alone), but the likely consequence is that evolution proceeds a little faster than predicted. This would lower the age to 2–3 Gyr, too young to be constrained by the Galaxy’s age.

Furthermore, no matter what formal age emerges from a theory, there seems to be no way to assure that EG Cnc or any other particular star is truly of great age, since the CV could have been born recently with a brown dwarf secondary. In Figure 7 we compare the mass-radius relation of the secondary in an old, whittled-down CV (HRP) with that of a single brown dwarf (Burrows et al. 1993). Also shown is the representation

[3] Paczynski (1981) and Paczynski & Sienkiewicz (1981) first applied this to CVs and demonstrated the need for a minimum period. RJW calculated the secondary’s changing radius, and that calculation has been repeated several times since (Rappaport et al. 1983; Nelson, Rappaport, & Joss 1986; HRP).

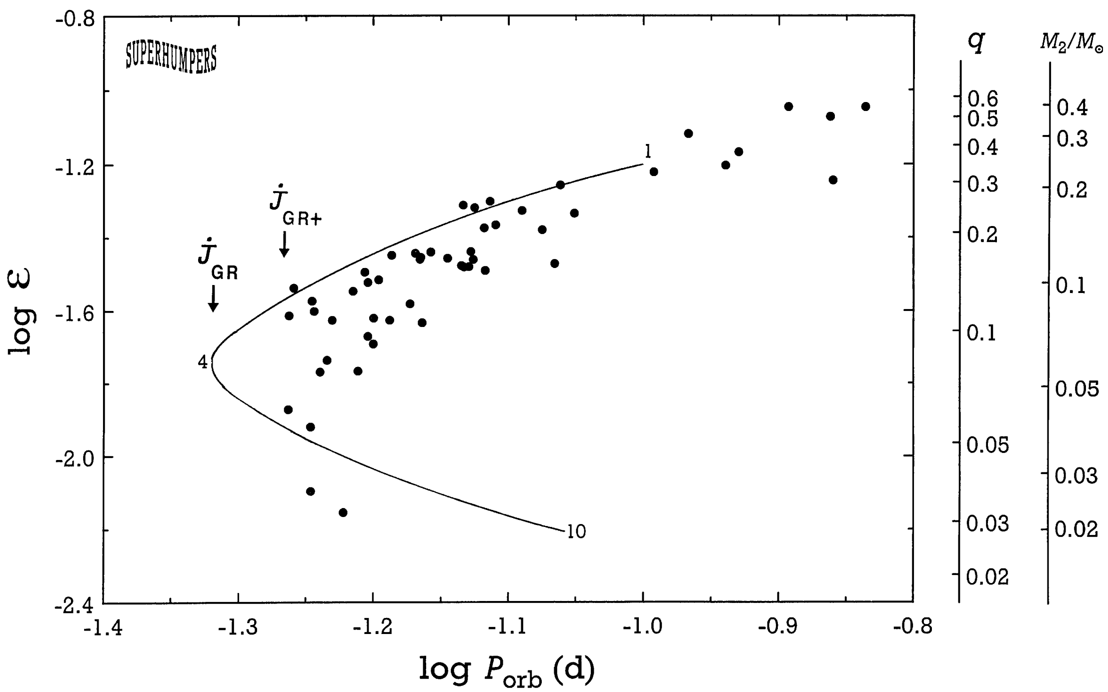

FIG. 6.—The ϵ-P relation compared with a prediction based on the mass-radius relation of HRP, a pure GR model. Numbers on the theory curve denote model ages in Gyr. All GR models reach a similar minimum P_{orb}, produce a similar bullet-shaped curve, and require about the same evolution time. The arrows indicate the minimum P reached in a GR model, and in a model with 50% greater $\dot{J}$.

of the lower main sequence by equation (4). The radii differ by less than 25% across the relevant mass range, and this seems too small to distinguish by observations, or to rely on as a firm difference between the theories. Thus we do not (yet) see sufficient basis for separating truly old CVs from young ones who just happen to have puny secondaries.[4]

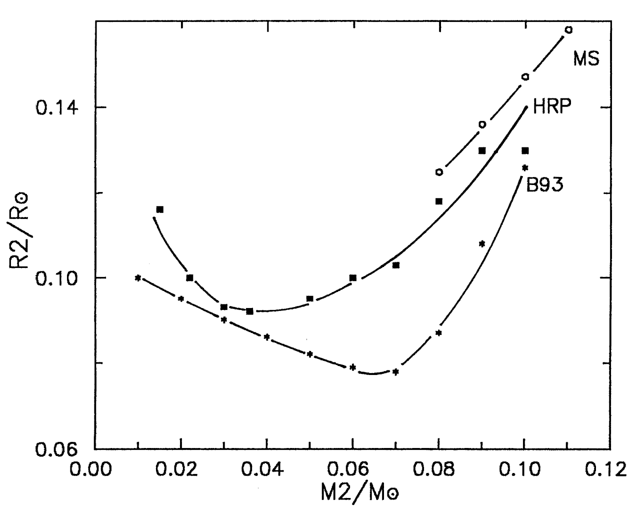

FIG. 7.—Comparison of the secondary's mass-radius relation in a whittled-down CV model (HRP) with the main-sequence relation of eq. (4) and a theoretical relation for brown dwarfs (Burrows et al. 1993).

The absence of known stars with $P > 0.06$ days and light secondaries is more likely to be due to their true rarity, or to selection effects discriminating against their discovery. We shall return to this subject in § 6.

5. INDIVIDUAL STARS

Now we turn to discussing the individual traits of stars of low ϵ. We omit two stars that are somewhat low for their period, V603 Aql and CP Pup, because they are both fast classical novae. Such stars are thought to arise from unusually massive white dwarfs, and a high M_1 (say 1.1–1.2 $M_\odot$) would fully explain their somewhat low ϵ. We discuss primarily the four stars with lowest ϵ(<0.013).

5.1. WZ Sge, EG Cnc, AL Com

These three are very similar stars. All show quiescent spectra dominated by the white dwarf, indicating an intrinsically faint contribution by accretion light ($M_v \sim 12$–13). Their very long

[4] Indeed, the low space density discussed by P84 and in § 6 suggests that there *are* no very old CVs.

recurrence periods (20–33 yr) also testify to a very low accretion rate.

5.1.1. WZ Sagittae

WZ Sge has a vast history in the literature. Greenstein (1957) first recognized the dominance of the white dwarf in the spectrum, although the significance of this result was not fully appreciated for many years.[5] Extensive studies appeared in the 1960s and 1970s (Krzeminski & Kraft 1964; Krzeminski & Smak 1971; Warner & Nather 1972; and many others). Perhaps the most provocative result was the upper limit $K_1 < 38$ km s^{-1} set by Krzeminski & Kraft (1964), implying an extreme mass ratio. Gilliland, Kemper, & Suntzeff (1986, hereafter GKS) measured $K_1 = 49$ km s^{-1} and inferred $M_1 = 1.1\ M_\odot$, $M_2 = 0.11\ M_\odot$ by assuming the secondary to be on the main sequence. Smak (1993) combined the GKS data with constraints from photometry and the spectroscopic s-wave and estimated $M_1 = 0.45\ M_\odot$, $M_2 = 0.06\ M_\odot$. Smak and GKS both assumed that $K_1 = 49$ represented the true dynamical motion of the white dwarf.

Confusion on the value of M_1 continues to reign. Ultraviolet spectroscopy (Sion et al. 1995; Cheng et al. 1997) favors a low white-dwarf gravity, but also a high rotation speed, which, if associated with the observed 28 s pulses, requires a fairly high $M_1 (>0.6\ M_\odot)$. Very high values ($>1.1\ M_\odot$) are probably forbidden by the low X-ray temperature and the low pressure broadening. These clues are not easily reconciled, but perhaps the most likely solution is a white dwarf of moderate mass (0.6–1.0 $M_\odot$), with rapid rotation somewhat mimicking low pressure broadening.

It is very unlikely that the GKS measurement of $K_1 = 49$ km s^{-1} represents the true dynamical motion of the white dwarf. The phase of this $v_{\rm rad}$ variation conflicts with the eclipse phase by 0.12 ± 0.02 cycles, a very serious flaw. And the UV measurements, using the C I lines that form high in the white dwarf photosphere, yield a limit $K_1 < 20$ km s^{-1} (E. M. Sion 1998, private communication). The latter should be more reliable than the traditional (and GKS) method of using the wings of disk emission lines. So severe a limit places strong constraints on a binary where an eclipse certifies high inclination. The effect of $K_1 < 20$ on $q(i)$ is shown in Figure 5 of Ritter & Schroeder (1979) and demonstrates $i = 81^\circ \pm 3^\circ$ and $q < 0.05$. The implications for M_2 are shown more explicitly in Figure 8. Possible choices for K_1 produce one family of curves, and possible choices for the disk radius produce another, using the observational constraint that $v \sin i = 720$ km s^{-1} at the edge of the disk. A traditional choice for the minimum disk radius is that of the zero-viscosity disk calculated by Lubow & Shu (1975); this is $(0.55–0.64)R_1$ in the relevant range of q. The maximum disk radius is not known, but a conservative limit is probably $\sim 0.8R_1$ (because one must leave some room to expand during outburst). So we will adopt $R_{\rm disk} = (0.6–0.8)R_1$. Finally, the secondary must be able to eclipse the bright spot for 164 s every orbit (Robinson, Nather, & Patterson 1978), which eliminates very small secondaries. The shaded region in Figure 8 shows the most likely stellar masses, with $M_1 = 0.77 \pm 0.20\ M_\odot$ and $M_2 = 0.024 \pm 0.009\ M_\odot$. The admissible range in q is 0.02–0.04.

This conclusion is not particularly sensitive to the less certain of our assumptions (those concerning the bright spot). Basically the value of K_1 in an edge-on binary of extreme mass ratio and $P = 1.36$ hr is given by 480 km s^{-1} $q(M_1/0.7\ M_\odot)^{1/3}$, so the observed limit on K_1 implies a hard limit $q < 0.042(M_1/0.7\ M_\odot)^{-1/3}$ even if no information about the bright spot is used.

5.1.2. EG Cancri

Much less is known about EG Cnc. At least 80% of the 4000–7500 Å light in quiescence comes from the white dwarf. In the range 6000–9000 Å, the flux distribution remains blue and featureless, leading to a limit of $M_v > 16.5$ for the secondary (Patterson et al. 1998a). This is very faint, but still not quite faint enough to firmly rule out a main-sequence star (from the M_2-M_v relations deducible from C98, we estimate $M_2 < 0.10\ M_\odot$, whereas our eq. [5] predicts $M_2 = 0.11\ M_\odot$).

In the distribution of Figure 6, it appears easier to reach EG Cnc by evolution along the curve shown than by a downward vertical plunge from the main cluster. Thus, with the lowest known ϵ, it might be considered the best candidate for having survived period bounce. On the other hand, WZ Sge has both a low ϵ and a very severe independent constraint on q (Fig. 8), so both stars are good candidates.

5.1.3. AL Comae Berenices

AL Com resembles WZ Sge in many respects, but its credentials as a period bouncer are only fair. A good radial velocity study might change this, but without eclipses to constrain inclination and with ϵ only moderately low, its credentials will perhaps never be excellent. Nor is there yet a compelling argument from the flux distribution. For our favored distance of 250 pc, the $K = 16.0$ measurement of SHM implies that M_K could be as bright as +9. This is not sufficiently faint to exclude a main-sequence secondary.

5.2. DI Ursae Majoris

The odd star in the collection is DI UMa. This is one of the most frantic dwarf novae in the sky, with normal eruptions

[5] And not fully remembered for many more years. There have been recent "discoveries" that many CVs are intrinsically faint (comparable, say, to field white dwarfs). Actually this traces back to Greenstein's work and was discussed extensively in reviews (P84; Smak 1984; Sion 1986; Warner 1987) and in dozens of analyses of individual stars (U Gem, Z Cha, OY Car, WZ Sge, HT Cas, etc.) in the 1980s, when it became widely realized that many low-$\dot{M}$ CVs in quiescence are dominated by the white dwarf at optical and near-UV wavelengths.

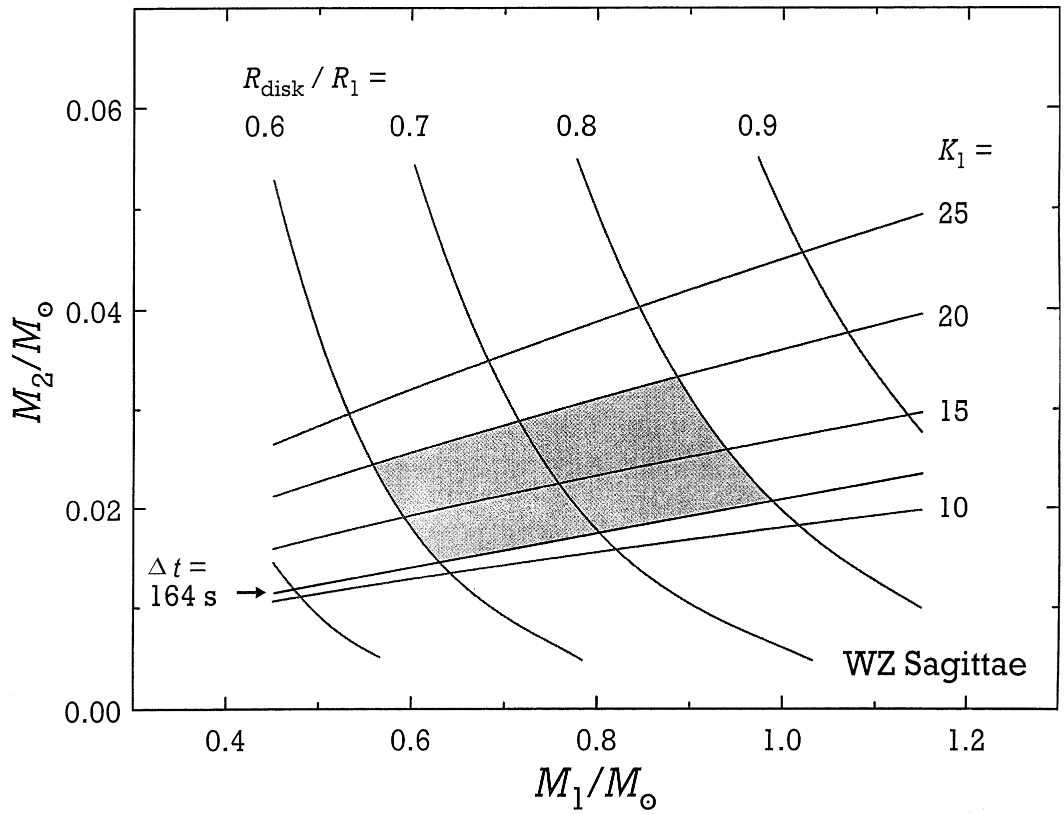

FIG. 8.—Constraints on M_2 and M_1 in WZ Sge. Curves are labeled with values of K_1 in km s^{-1}, and R_{disk}/R_{1L}, where R_{1L} is the radius of the primary's Roche lobe. The latter is probably in the range 0.6–0.8. A lower limit to M_2 is set by the requirement that the secondary must eclipse the bright spot for 164 s. Acceptable values of M_2 and M_1 are in the shaded region.

every 8 days and superoutbursts every 35 days (Kato et al. 1996a; Fried et al. 1998). Therein lies its oddness, because so much accretion light gives the star a time-averaged M_v around +7—more luminous than the WZ Sge class by a factor ~50.

Do we understand why this star should have low ϵ? No. Its comrades in eruption characteristics, the ER UMa subclass of the SU UMa stars, have ϵ normal to slightly high (Thorstensen et al. 1997). A massive white dwarf (1.2 $M_\odot$) would explain it, but there is no evidence for that.

A ready excuse is available in the possibility of mass-transfer cycles. While most short-period CVs are intrinsically faint, there are eight that are far more luminous, indicating an accretion rate 10–100 times higher. These are the novalike variable BK Lyn, three classical nova remnants (V1974 Cyg, CP Puppis, T Pyx), and the four known ER UMa stars (ER UMa, RZ LMi, Vl159 Ori, DI UMa). So high a dispersion at a fixed period presents a serious problem. But it can be understood if evolution proceeds through cycles of high and low mass transfer, as suggested many times previously (Wu et al. 1995; King 1995; King et al. 1996). The idea is that ϵ really does signify q, but a few decades is an insufficient time baseline to deduce a true $\langle M_v \rangle$ or $\langle \dot{M} \rangle$, because of long-term cycles.

Two facts seem noteworthy about these excessively bright stars:

1. They are rare. Probably none are within 200 pc, where ~20 "normal" dwarf novae live.
2. About half of them (at much greater distances, so not truly half in the actual space density) are in classical novae that have erupted in the last 60 yr.

From these facts we speculate that the mass-transfer cycle may be associated with classical nova eruptions (possibly related to the well-known "hibernation scenario" suggested by Shara et al. 1986 and Kovetz, Prialnik, & Shara 1998). Over an interval ~1%–5% of the cycle time, a short-period CV surges to high $\dot{M}$ and therefore high accretion luminosity. Classical nova eruptions tend to occur then. As applied to DI UMa, the idea is that it spends most of its life as an ordinary WZ Sge star, as suggested by the low ϵ, but we have caught it during an upward surge in accretion.

It is also possible that high M_1 alone explains the entire effect. X-ray and ultraviolet observations (or better yet, a classical nova eruption) are needed to assess this possibility.

6. FROM CENSUS TO SPACE DENSITIES

So far we have discussed a census of known CVs, mainly the subset with superhumps. But most CVs are dwarf novae, most dwarf novae are of the SU UMa type, and all SU UMa stars show superhumps. So at least for studying the behavior at short period, our main purpose, this is essentially the dominant group. Now we study the period distribution and space density of CVs, supplementing the earlier discussion of P84 (§ V of that paper).

6.1. Comparison with Theory

The theory generating the curve of Figure 6 also predicts the distribution of CVs along the curve. Stars injected at the upper right evolve at a certain rate, calculated from the prescription for angular momentum loss. The most commonly used prescription is the Verbunt & Zwaan (1981) formula for magnetic braking longward of the 2–3 hr period gap and gravitational radiation shortward of the gap. This produces far more binaries at very short $P_{\rm orb}$, since the evolution lifetimes lengthen there. Kolb (1993) estimated that 99% of CVs should have $P < 2$ hr, and 70% should have passed period bounce. This is *prima facie* inconsistent with the data of Figure 1, unless selection effects discriminate very heavily against discovery of the short-period stars.

6.2. Selection Effects

There are selection effects in any roster of variable stars. The important ones here are selection due to frequency of variability (rare erupters are harder to discover), selection due to magnitude (fainter stars are harder to discover), and selection due to inability to detect the superhumps (because of insufficient hump amplitude or frequency resolution).

The last of these is unlikely to be important. Essentially all SU UMa stars now receive photometric coverage in superoutburst, and the humps are always easy to detect. It is common for observational campaigns to last ∼10 days, which would be sufficient to reveal ϵ as low as 0.003. So the absence of stars in the lower right part of the diagram is unlikely to arise from the third bias.

Magnitude selection certainly matters. Some dwarf novae reach only 15th magnitude in outburst, and we have very little idea what fraction of erupters we manage to identify at this flux level. Surely it must be quite low. Discovery methods improve sharply at about magnitude 12–13, and even more sharply at magnitude 10, since that is the appropriate limit for legions of visual observers using large binoculars and small telescopes to scan essentially the whole sky for comets and novae. Magnitude 10–12 is also the approximate limit for photographic plate collections that cover truly wide fields (with the “astrographs” of yesteryear). Thus it seems very unlikely that the sky contains large numbers of undiscovered dwarf novae reaching 10th magnitude. According to equation (1), dwarf novae with $P \sim 1.3$ hr reach $M_v \sim +5.4$ in outburst. This suggests no great undercount out to 80 pc. But there is just one candidate period bouncer (WZ Sge) in this volume, giving a formal $N \sim 10^{-6}$ pc^{-3}. This falls short of Kolb’s prediction by a factor of more than 20.

The only possible full rescue of the hypothesis is that the stars hardly ever erupt. Since we have been looking and photographing for about 100 yr, we can claim an arbitrarily low efficiency k of eruption discovery by supposing a recurrence time ∼100/k yr. To satisfy Kolb’s prediction we require $k <$ 0.05 and hence a recurrence time exceeding 2000 yr. This is not firmly excluded by observation, although it does require believing in a new class of stars, probably none of which have been certified yet (even EG Cnc and WZ Sge, whose credentials are otherwise good, erupt every 30, not 2000, yr).

6.3. Constraints from Flux-limited Surveys

For stars that can be discovered by some means other than variability, it is possible to be more quantitative. There are basically four flux-limited surveys that cast a net sufficiently wide and sufficiently well-defined that we can use the results to constrain the space density of CVs. These are the Palomar-Green survey for blue stars ($U - B < -0.46$) at high Galactic latitude (Green, Schmidt, & Liebert 1986), the various all-sky surveys for strong 2–10 keV X-ray sources (Piccinotti et al. 1982; Silber 1991), the *Einstein* Medium Sensitivity Survey for 0.2–4.0 keV X-ray sources (Stocke et al. 1983), and the *ROSAT* All-Sky Survey for 0.1–2.8 keV X-ray sources (Bade et al. 1998). Except for the latter, all of these were discussed by P84. We repeated that analysis, incorporating the *ROSAT* results and all stars newly discovered since 1984, and found much the same result as expressed in Table 5 and Figure 10 of P84: the total space density of active CVs is ∼10^{-5} pc^{-3}, with about 60%–80% below the period gap.

For the very low $\dot{M}$ case, our major interest, the three WZ Sge stars just described will illustrate the point. We estimate $M_B = 11.5$ for these stars, and their $U - B$ colors (-0.7 to -1.0) easily satisfy the Palomar-Green threshold ($U - B < -0.46$) for detection and spectroscopy. The survey’s average magnitude threshold for completeness is $B = 16.2$, implying that such stars should be detected to 87 pc. With 10,700 square degrees searched to this distance, and assuming an incompleteness of 16% (Green et al. 1986), we obtain an effective search volume for such stars of 3×10^5 pc^3. Among the 81 CVs and CV candidates detected in the survey, none are certifiably of the WZ Sge type (white-dwarf absorptions plus emission lines, the obvious and strong signature). But examination of the published and unpublished spectra shows four that could be considered candidates, implying that the space density of such stars is $\lesssim 10^{-5}$ pc^{-3}.

These estimates will not greatly change unless a new and dominant population of binaries is found, which manages to avoid detection by all of the common criteria: blue color, X-rays, and variability. This is formally possible but seems unlikely. Why? Because low-$\dot{M}$ CVs, the class of maximum interest for this study, tend to be found by all three methods: blue color through the underlying white dwarf and through the confluence of Balmer emission lines in the *U* band, X-rays through the typically high L_X/L_{opt} ratios characteristic of such stars, and variability through dwarf-nova eruptions. If a large population of this type exists, it must be very craftily concealed to avoid all the radar screens of detection, and must not much resemble any of the stars we call “cataclysmic variables.”

6.4. Comparison with Theory, Revisited

In P84 we estimated that 20%–33% of CVs are above the period gap, and if our accounting for selection is somewhat erroneous it could be as low as 10%. Below 10%, we find it too hard to understand why all those short-period binaries are eluding detection. Thus we believe that the data are inconsistent with Kolb’s prediction by a factor of more than 10.

How can we fix the theory? Well, it can be improved by nibbling at the margins. It assumes mass-transfer rates predicted by the Verbunt & Zwaan (1981) formula for magnetic braking, and these could be too high by a factor of as much as 3. The time in the detached state between 2 and 3 hr could be underestimated by a factor of ∼2. Some tinkering with the mass-radius relation at low mass could help. If these uncertainties move in the same direction, then it is remotely possible that they might combine to reach consistency with observation.

But we doubt it. There is another route to consistency that requires no such agility, but does require an extra hypothesis. This is the solution we previously advocated in discussing this issue (P84, where it appears as “the problem of the dead novae”). Namely, to accept the shortfall and prevent CVs from reaching unacceptably high space densities by *destroying* them. Opportunities for self-destruction arise when the secondaries are threatened with the loss of thermal equilibrium, which can occur near $P = 3$ hr and again at 1.3 hr.

7. SEARCHING FOR PERIOD BOUNCERS

To summarize, we regard low M_2 as the principal credential of period-bounce candidates; in practice, we still have no way to constrain M_2 directly, but spectroscopy and photometry can constrain the mass ratio *q*. And, if the simple theory of disk precession peddled here is accepted, so can the measurement of ϵ through studying superhumps. Application of this method to all superhumping stars yields no incriminating errors and produces a “main sequence” in Figure 4 that quantitatively agrees with the actual main sequence. Thus, there are fairly good grounds for believing that the four stars of very low ϵ have very low *q*. Two are likely to have $q \simeq 0.03$, from which we estimate $M_2 \sim 0.02\ M_\odot$. Three of the four stars are members of the WZ Sge class.

What exactly is this class? It is basically a subset of the SU UMa stars, distinguished by showing very rare and primarily long eruptions. So far, however, no observational distinction has emerged which clearly marks this class as separate from its nearest relatives, the more sluggish of the “normal” SU UMa stars. So probably it is just a name by which to label one extreme of dwarf nova behavior. What does impress us, though, is the great faintness of accretion light in these three stars. For WZ Sge we estimated $M_v = +12.2$ at quiescence and $\langle M_v \rangle = +11.5$ averaged over an eruption cycle (Patterson et al. 1996). We estimated these quantities at $+12.6$ and $+11.2$ in EG Cnc (Patterson et al. 1998a) and $+12.8$ and $+11.7$ in AL Com (Patterson et al. 1996). These are the three most intrinsically faint CVs known.

If gravitational radiation powers mass transfer and the dispersion in M_1 is low, then near minimum period, roughly $\dot{M} \propto \dot{J} \propto M_2^2 \propto q^2$. This implies that the three WZ Sge stars, with *q* about a factor 2.5 below the main cluster of short-period stars, should have accretion rates about a factor of 6 lower. That seems just about right; in Figure 2 we see that WZ Sge stars average about 2 mag fainter than the average SU UMa stars.

It would be wonderful to find more stars near the lower branch of the curve in Figure 6! Many other candidate WZ Sge class members have been proposed (Bailey 1979a; Downes & Margon 1981; O’Donoghue et al. 1991; Howell, Szkody, & Cannizzo 1995). But for most stars on these lists we still lack knowledge of the orbital period, and even knowledge of the star’s outburst frequency and amplitude. So there is much work to be done.

We did not find support for the proposal that dwarf novae of large amplitude are period bouncers (HRP). Howell et al. (1995) proposed 28 such stars as candidates. Only two stars on that list, WZ Sge and AL Com, are among our four candidates; and the seven others with *P* and P_{sh} known (WX Cet, VY Aqr, AK Cnc, SW UMa, LL And, HV Vir, UV Per) have on average normal ϵ. Since dwarf novae in outburst are fairly good standard candles, large amplitude correlates with intrinsic faintness at quiescence. But many other properties go along with intrinsic faintness: strong emission lines, cool accretion disk, high L_X/L_{opt}, low frequency of outburst, flux domination by the white dwarf, and low temperature of the white dwarf. Among these many correlatives, there appears to be no reason to regard amplitude as anything more than a useful clue. The really important point is *low average accretion rate.*

8. WZ SAGITTAE AS A “DEATH STAR”

WZ Sge is certainly the most interesting of these stars, since observations establish low *q*, low ϵ, and low $\langle \dot{M} \rangle$. Recent estimates suggest $\langle \dot{M} \rangle \simeq 2 \times 10^{15}$ g s^{-1} (Smak 1993; Patterson et al. 1996). Since we have reason to believe $M_2 \simeq 0.02\ M_\odot$, we estimate that the secondary is being stripped on a timescale $M_2/\dot{M} = 7 \times 10^8$ yr.

This is an interesting number, ∼10% of the Galaxy’s age. It suggests that the secondary is being evaporated fairly rapidly, sufficient to kill off moderately old CVs and give a welcome factor of 10 respite from the space-density problem discussed above. That would be sufficient to solve the problem.

Near minimum period, a pure GR theory actually makes *three* predictions which are not quite satisfied:

1. It predicts a minimum period of 1.1 hr, not 1.3 hr as observed.
2. It floods the sky with too many low-$\dot{M}$ stars.
3. It yields a predicted $\epsilon(P)$ relation with a fairly sharp cusp at minimum period, as seen in Figure 6. The available data, though sparse, suggest a rather broad cusp—or even just a vertical fall.

The extent of these failures is debatable. We regard only the second as (almost) secure. The first could possibly be explained by tinkering with the mass-radius relation. And the data are still too sparse to establish the third. Nevertheless, it is interesting to note that all three failures can be explained with a single hypothesis: that there is another source of angular momentum loss. Let us suppose for simplicity that it is equal to half the approximately constant $\dot{J}_{\rm GR}$ for prebounce CVs. This increases the total $\dot{J}$, which increases the minimum period by ~15% since $P_{\rm min} \propto (\dot{J}/\dot{J}_{GR})^{0.34}$ (Paczynski 1981). This solves the first problem, as shown by the movement of the arrow in Figure 6.

Evolution proceeds faster for prebounce stars, so this helpfully speeds stars through their prebounce lives. After the star falls out of thermal equilibrium near 0.08 $M_\odot$, $\dot{J}_{\rm GR}$ drops because $\dot{J}_{\rm GR} \propto q^2$. But by hypothesis, the extra component of $\dot{J}$ remains constant. This means that postbounce stars will continue to lose mass at a moderately high rate, not at the rapidly declining rate of a pure-GR theory. This soon kills off the CV and thereby solves the second problem.

The sharpness of the cusp in Figure 6 is a measure of how far the secondaries are from thermal equilibrium. For $\dot{J}$ exceeding $\dot{J}_{\rm GR}$, the stars are farther from equilibrium and fall more rapidly in the $\epsilon(P)$ diagram, causing a broader cusp as observed. This solves the third problem.

So that is quite a nice reward from one hypothesis. We regard it as a promising possible solution to the problems discussed here.

This scenario probably will not cannibalize the secondary entirely. We use the word "death" because the binary should no longer be cataclysmically variable. But when the secondary reaches a mass ~0.002 $M_\odot$, it may become more planetlike (supported by Coulomb pressure) and then fall in radius with further mass loss. That is the expected final state of a CV: a white dwarf with a planet. Of course, there is plenty of room in the sky for objects like that!

9. SUMMARY

1. We discuss the observational evidence for cataclysmic variables in a very late phase of evolution, near the 1.3 hr "period minimum" for H-rich stars. The dwarf novae are a particularly suitable subgroup, since they dominate the census at short period.

2. The most telling signature of a star's evolutionary status is M_2, the mass of its secondary. A spectroscopic measure of M_2 is very difficult even under favorable circumstances and essentially impossible when M_2 gets very low. But it can be probed by the fractional period excess ϵ of superhumps. The empirical $\epsilon(P)$ relation provides strong evidence that secondaries follow a main-sequence mass-radius relation in most short-period CVs, but deviate at the shortest periods.

3. The deviants must have secondaries of low mass and "oversized for their mass," which makes it plausible that they reached that state by losing thermal equilibrium as they dropped below ~0.08 $M_\odot$. The two most extreme stars, EG Cnc and WZ Sge, appear to have $q \sim 0.03$ and therefore $M_2 \sim 0.02\ M_\odot$.

4. Three of the four principal deviants are WZ Sge–type dwarf novae and, indeed, are the most intrinsically faint stars among all CVs of known orbital period. This is consistent with the ascription of very low mass ratios, since roughly $\dot{M} \propto q^2$ in a short-period system evolving by gravitational radiation.

5. As suggested by the curve in Figure 6, period bounce is an attractive explanation for these stars. However, they are not very common, representing probably less than 30% of all CVs, and their confinement to $P < 1.5$ hr shows that *there are no stars yet studied that have evolved very far past period minimum.*

6. The best constraints on space density are still furnished by large-area surveys for blue stars and X-ray sources. We repeated the study of P84 and found similar results: the total space density of active CVs is ~10^{-5} pc^{-3}, with ~75% below the period gap. These estimates will not greatly change unless a new and dominant population of binaries is found, which manages to avoid detection by all of the common criteria: blue color, X-rays, and variability.

7. Such a population might exist, but would require careful engineering. With secondaries estimated at ~0.02 $M_\odot$, WZ Sge and EG Cnc might be considered prime examples. Yet their $U - B$ colors in quiescence (−0.7; Krzeminski & Smak 1971; Patterson et al. 1998a) are easily blue enough to be detected by the Palomar-Green survey. So it is difficult to envision how large numbers of nearby CVs can avoid detection in surveys like this.

8. In a popular theory of CV evolution, stars go through their lives with $P > 3$ hr in ~10^8 yr, and then populate the short-period regime in enormous numbers (99% of the total; Kolb 1993) as they continue to evolve. Barring discovery of

a new class of exotic variables, present observations appear to be inconsistent with this prediction.

9. It would be nice to find a way to destroy CVs before they reach the very high space density, and very high pileup at short period, predicted by theory. Such an opportunity may be presented by the secondary's loss of thermal equilibrium at $P = 3$ hr and 1.3 hr.

10. Searches for stars that may have evolved far beyond period bounce would be very desirable. Any bona fide WZ Sge star with $P > 0.06$ days is an excellent candidate (RZ Leo is the only one known to date). It would also be good to consider stars that now elude identification as CVs, because there is no known eruption, the continua are insufficiently blue, etc. White dwarfs with infrared excess and/or emission lines are promising targets for study.

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