THE ASTROPHYSICAL JOURNAL, 385:621–629, 1992 February 1

A MECHANISM FOR ORBITAL PERIOD MODULATION IN CLOSE BINARIES

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Received 1991 April 19; accepted 1991 August 6

ABSTRACT

Some eclipsing variables are observed to undergo orbital period modulations of amplitude $\Delta P/P \sim 10^{-5}$ over time scales of decades or longer. These modulations can be explained by the gravitational coupling of the orbit to variations in the shape of a magnetically active star in the system. The variable deformation of the active star is produced by variations in the distribution of angular momentum as the star goes through its activity cycle. This mechanism typically requires that the active star be variable at the $\Delta L/L \simeq 0.1$ level, and be differentially rotating at the $\Delta\Omega/\Omega \simeq 0.01$ level. The torque needed to redistribute the angular momentum can be exerted by a mean subsurface magnetic field of several kilogauss.

Subject headings: binaries: eclipsing — stars: magnetic fields

1. INTRODUCTION

Eclipsing variables are excellent laboratories for studying a wide variety of processes in stellar astrophysics. These objects offer probes of tidal dissipation, mass transfer or loss, angular momentum transfer or loss, magnetic activity, and stellar evolution. Clearly, their usefulness extends far beyond their textbook role in the determination of stellar masses and radii. This usefulness largely derives from the fact that precision measurements can be made. The careful timing of eclipses can reveal orbital period changes of order a part in 10^5–10^6 because deviations from an assumed ephemeris can build up over many orbits, and many systems have observational records spanning decades or more. These observational records reveal a surprising result: systems which show period changes of alternating sign (orbital period modulations) are common. In this paper I propose a model which explains these orbital period modulations as a consequence of magnetic activity in one of the stars in the binary. In this model the distribution of angular momentum in the active star changes as the star goes through its activity cycle. Variations in the distribution of angular momentum produce variations in the oblateness of the star. These changes are communicated to the orbit by gravity, changing the orbital period. The observed amplitude of period modulation, $\Delta P/P \sim 10^{-5}$, can be produced if the active star is variable at the $\Delta L/L \simeq 0.1$ level, and differentially rotating at the $\Delta\Omega/\Omega \simeq 0.01$ level. Both this level of variability and degree of differential rotation are common in the classes of binaries which show orbital period modulation (Hall 1990).

The most striking example of orbital period modulation is Algol. The observational record (Söderhjelm 1980) extends back two centuries, which is more than 25,000 cycles of the 2.87 day orbit. A diagram of the time dependence of the difference between observed times of eclipses and those predicted by a constant period ephemeris (the $O-C$ diagram) shows a feature (the great inequality) with a full amplitude of 0.3 days and, if it is assumed recurrent, a period of roughly 200 yr. In addition, there is a feature with a full amplitude of 0.06 days and a recurrence time scale of 30 yr. Several cycles of the 30 yr modulation are apparent in the $O-C$ diagram. Both of these features correspond to orbital period changes of full amplitude $\Delta P/P \simeq 3 \times 10^{-5}$. Orbital period changes of this amplitude are seen in other Algols as well (Kreiner & Ziołkowski 1978; Hall 1989). Orbital period modulations of amplitude $\Delta P/P \sim 10^{-5}$ and decade time scale are also seen in RS CVn's (Hall & Kreiner 1980), V471 Tauri (Skillman & Patterson 1988), W UMa stars (Glownia 1986), cataclysmic variables (Pringle 1975; Warner 1988), and other systems (Kreiner 1971).

A variety of theoretical models have been proposed to explain these orbital period modulations. The simplest and most conservative models explain the observed period changes as due to apsidal motion or a distant, unseen companion. Both of these hypotheses fail in a crucial way. Third bodies and apsidal motion both require that the period modulations be strictly periodic, but the observations show that this is not the case. In addition, Van Buren (1986) has shown that the requirements that the third body be both massive enough to produce the observed modulation and faint enough to have escaped detection are often inconsistent, ruling out a distant third body as an explanation unless the body is quite exotic. Apsidal motion requires an eccentric orbit, and many systems which show period modulation are observed to have small, if any, orbital eccentricity. Circular orbits are expected in these systems because they are close enough for tidal interactions or mass transfer to rapidly damp out any eccentricity.

The decade time scale and lack of strict periodicity of the orbital period modulations suggests that they might be a manifestation of magnetic activity. A strong argument in favor of this hypothesis was presented by Hall (1989), who found that of the 101 Algols he studied, all 31 which showed orbital period changes of both signs contained at least one star with a convective envelope. Since all of the stars in the Algols should be rapidly rotating, the presence or absence of a convective envelope determines whether or not all of the ingredients of a magnetic dynamo are present. Orbital period modulations only occur in those systems which possess all of the ingredients of a dynamo. Additional evidence supporting the interpretation of orbital period modulation as a manifestation of magnetic activity is given by Hall (1990).

The magnetic fields of the Sun and other stars are thought to be the result of magnetic dynamo action occurring in their convective envelopes or at the interface between their convective envelopes and radiative cores. Magnetic activity results from the interplay between differential rotation and cyclonic convection. In this picture (see Parker 1979, Figs. 18.2–18.5) an initially poloidal magnetic field is sheared by differential rotation, producing a toroidal subsurface field. Toroidal field is carried to the surface of the star by convection, and given a twist because the convection is cyclonic. The twist, provided by the Coriolis acceleration, turns toroidal field back into poloidal field and completes the cycle. The action in dynamo models is in the dynamics of the subsurface field; the surface magnetic activity is a sideshow. Orbital period modulation can be a powerful tool for studying stellar magnetic activity because the mechanism that produces the period change depends on the magnitude of the subsurface magnetic field. The observed amplitude and period of orbital period modulation can be used to determine the mean subsurface field strength (see eq. [33] below). These measurements, not possible by any other technique, will provide important constraints on dynamo models of magnetic activity.

The history of solar magnetic activity (Baliunas & Vaughan 1985) shows that the period of the sunspot cycle is 11 yr on average, but the interval between successive solar maxima can vary from 8 to 15 yr. The sunspot record also shows evidence of cycles of much longer duration. By analogy to the Sun, magnetic activity should be expected to produce regular, but not strictly periodic, changes in an active star, and several cycles of different duration may be present. These qualitative expectations are an excellent description of what is seen in the $O-C$ diagram of Algol.

The identification of a mechanism for coupling the orbit to changes in an active star is a crucial problem for any model that proposes magnetic activity as the underlying cause of orbital period modulations. A direct torque on the orbit due to the rocket effect produced by anisotropic mass loss was considered and rejected by DeCampli & Baliunas (1979). Tidal torques have been suggested by several authors, but the magnitude of the tidal torque (Zahn 1977, 1989) is orders of magnitude too small to transfer the necessary angular momentum (Applegate & Patterson 1987) unless the accepted tidal friction calculations are incorrect (Tassoul 1987, 1988).

The problem of the lack of a suitable torque to couple changes in the active star to the orbit was solved when Matese & Whitmire (1983) and, independently, Applegate & Patterson (1987) realized that a coupling torque was not necessary. These authors realized that the orbital period would be changed at constant orbital angular momentum if the radial part of the gravitational acceleration varied. This would occur if the quadrupole moment of the active star varied through the activity cycle. The mechanism is simple. The orbital period is $P = 2\pi a/v$, where a is the separation and v is the relative velocity. The angular momentum is $J = \mu v a$, where μ is the reduced mass. The velocity of a circular orbit is related to the separation and gravitational acceleration g by $v^2 = ag$. Combining these gives Kepler's third law for a circular orbit in the unfamiliar form $P = 2\pi(J/\mu g^2)^{1/3}$, which gives $\Delta P/P = -\frac{2}{3}(\Delta g/g)$ at fixed orbital angular momentum. The gravitational acceleration will vary if the shape of the star varies; this shape variation is measured by the change of the quadrupole moment of the star.

The recognition of the role of gravitational quadrupole coupling in producing orbital period changes transferred the problem from the orbit to the active star: how are the shape changes produced? A magnetic field can produce recurrent deformations in an active star by distorting the star away from a state of fluid hydrostatic equilibrium, or by causing transitions between different states of fluid hydrostatic equilibrium. In the various models in the literature (Matese & Whitmire 1983; Van Buren & Young 1985; Applegate & Patterson 1987; Warner 1988) the authors have all assumed that the magnetic field deforms the star by distorting it away from a fluid equilibrium shape. This mechanism requires that the magnetic field make a nontrivial contribution to hydrostatic equilibrium, which requires a large magnetic field. These models were dealt a serious blow when Marsh & Pringle (1990) argued that the energy required to make the deformation was larger than what the luminosity of the star could provide. The Marsh & Pringle argument is straightforward. The energy required by the proposed distortions is of order $\Delta E \simeq (GM^2/R)(\Delta P/P)$, where M and R are the mass and radius of the active star. If this energy is supplied by the nuclear luminosity of the star, the modulation time scale t_{mod} must satisfy the inequality $t_{\text{mod}}/t_{\text{KH}} > \Delta P/P$, where $t_{\text{KH}} = GM^2/RL$ is the Kelvin-Helmholtz time of the active star. Marsh & Pringle noted that this requirement is not satisfied.

In their analysis Marsh & Pringle (1990) used an estimate of the magnetic field strength taken from Applegate & Patterson (1987). This estimate is rather crude. In § 2 below, I refine the estimate of the field strength needed to produce a given deformation, and give a detailed analysis of the energy and force requirements of distortions away from fluid hydrostatic equilibrium. I find that the Marsh & Pringle result, although not rigorous, is almost certainly correct, and I believe that distortions away from hydrostatic equilibrium are ruled out as possible mechanisms for producing orbital period modulations.

The mechanism which I propose to produce the period changes invokes the magnetic field to cause transitions between states of fluid equilibrium. The rotational part of the quadrupole moment of the active star reflects the distribution of angular momentum within the star. In particular, the quadrupole moment is most sensitive to the rotation rate of the outer part of the star. If the distribution of angular momentum changes as the star goes through its activity cycle, possibly due to the action of a magnetic torque, the rotational oblateness of the star will change, and this change will be communicated to the orbit by gravity, changing the orbital period.

This mechanism requires that the active star be variable, and that the period of the luminosity variation be the same as the period of the orbital period modulation. The physics of the variability is simple. During one phase of the cycle of angular momentum transfer the star is driven away from solid body rotation. This increases the rotational kinetic energy of the star because solid body rotation minimizes the energy for a given angular momentum. The luminosity of the star falls during this phase of the cycle because energy is removed from the heat flow and pumped into differential rotation. The luminosity of the star increases during the second phase of the cyle. During this phase the star approaches solid body rotation; this increases the luminosity because energy is taken out of differential rotation and returned to the heat flow. The luminosity variation predicted to accompany orbital period modulation has been detected in CG Cygni by Hall (1991), who finds that both the luminosity of the active star and the orbital period vary with a period of about 50 yr. This important result is a strong argument in favor of the model of orbital period modulation presented here.

The variation of the rotational kinetic energy required by the model will produce luminosity variations if the convective zone of the active star can respond fast enough. If it cannot, the variations in the heat flow will be damped, and luminosity variations will not be seen. In this case the convective zone acts like a low pass filter, damping a high-frequency signal. The response of a convective envelope to a variable energy input is an unsolved theoretical problem. Observationally, Hall's (1991) data implies that a significant fraction of the dissipated rotational kinetic energy is being radiated away instantaneously. The luminosity variation of CG Cygni is consistent with the variability required by the orbital period change, so the damping of the variability cannot be too large (the quantative meaning of this statement will be explored in a later paper). In addition, the relative phase of the luminosity and $O-C$ diagram variations are what is expected for a positive angular velocity gradient (see below and Hall 1991) and no damping by the convective zone. Qualitatively, damping should produce a phase shift between the energy input, measured by the $O-C$ variation, and the luminosity variation. This phase shift is not seen, implying that the damping cannot be too large. The convective zone in the active star in CG Cygni can respond in 50 yr.

The energy budget of orbital period modulation typically requires the active star to be variable $\Delta L/L \sim 0.1$ level, and to the differentially rotating at the $\Delta\Omega/\Omega \sim 0.01$ level. The magnitude of the torque needed to transfer the angular momentum implies that the mean subsurface magnetic field strength must be several kilogauss. For some reason, the magnetic dynamos in rapidly rotating stars in close binaries seem to like these numbers.

2. DEFORMATION MECHANISMS AND ORBITAL PERIOD CHANGES

2.1. *General Considerations*

The orbital period of a close binary will change in response to changes in the shape of one of the stars because the orbit is coupled to the internal structure of the stars by gravity. Regular, but not strictly periodic, shape changes may be a consequence of stellar magnetic cycles. Deformation mechanisms fall into two broad categories: distortions, which are mechanisms whereby the magnetic field distorts the star away from a figure of fluid hydrostatic equilibrium, and transitions, which are mechanisms whereby the magnetic field affects transitions between fluid hydrostatic equilibria. A simple example of a distortion is the expansion and contraction of an active star as the magnetic pressure varies. A simple example of a transition, to be developed in detail below, is a variable rotational oblateness produced by the redistribution of angular momentum by a magnetic torque. If the magnetic field producing a distortion is suddenly switched off, the star is left out of hydrostatic equilibrium and begins to oscillate in its adiabatic pressure modes. If the magnetic field producing a transition suddenly vanishes, nothing happens; the star is already in hydrostatic equilibrium. The only consequence of the field vanishing is that the shape of the star stops changing.

The energy required to deform a star in a close binary and change the orbital period of the binary is the sum of three contributions: the energy needed to make the deformation, the energy needed to stabilize the deformation if unbalanced forces have been produced, and the energy needed to change the orbit and rearrange the tidal deformations of the stars. The size of the deformation is measured by the ratio ψ/R, where ψ is the Lagrangian deformation (to be defined below) and R is the radius of the star; very roughly $\psi/R \sim \Delta P/P \sim 10^{-5}$. The active star is in hydrostatic equilibrium in its undeformed configuration, and the unperturbed orbit is circular. Hydrostatic equilibrium minimizes the energy with respect to adiabatic deformations, and a circular orbit is the minimum energy orbit with a given orbital angular momentum. Therefore, the energy needed to rearrange the orbit and the tidal deformations of the two stars is always second order in ψ/R and is negligible. If the star is deformed by distortion, the distortion can be made by displacing the normal modes of the star from their equilibrium positions. The potential energy of a harmonic oscillator is quadratic in the displacement of the oscillator, so the energy cost of making a distortion is of order $(\psi/R)^2$ and is negligible. Deformation by distortion leaves gravity and the pressure gradient out of balance. This unbalanced force must be balanced by the magnetic field, and the energy cost of stabilizing the distortion can be either first or second order in ψ/R depending on the nature of the distortion. The force required to stabilize the distortion is always first order in ψ/R because the mode obeys Hooke's law for small displacements. A deformation by transition, by definition, does not involve unbalanced forces, and does not require stabilization. In the particular model developed below, the deformation is made by transferring angular momentum to and from the outer layers of the active star. Since the star is probably differentially rotating, the angular momentum transfer has an energy cost which is first order in ψ/R, but with a small coefficient.

2.2. *Gravitational Quadrupole Coupling*

Consider a close binary system containing an magnetically active star, whose internal deformations will be treated in detail, and a companion star, which will be treated as a point mass. The mass and radius of the active star are M and R, and the two stars are separated by a distance a in circular orbits. The gravitational potential $\phi(\boldsymbol{x})$ outside of the active star is

$$\phi(\boldsymbol{x}) = -\frac{GM}{r} - \frac{3}{2} G Q_{ik} \frac{x_i x_k}{r^5} , \tag{1}$$

where x_i and x_k are Cartesian coordinates measured from the center of mass of the active star, the summation convention has been assumed, and deformation terms of higher order than quadrupole have been dropped. The quadrupole tensor Q_{ik} is the traceless part of the inertia tensor I_{ik},

$$Q_{ik} = I_{ik} - \tfrac{1}{3}\delta_{ik} \operatorname{Tr} I , \tag{2}$$

where Tr I is the trace of the inertia tensor. The inertia tensor is defined by

$$I_{ik} = \int dm x_i x_k = \int d^3 x \rho(\boldsymbol{x}) x_i x_k . \tag{3}$$

The binary systems under consideration are all very close. Accordingly, I assume that tidal friction has synchronized spin and orbit, circularized the orbit, and brought the rotational and orbital angular momenta into alignment. I take the $\hat{z}$-axis to lie in the direction of the angular momentum. I let the $\hat{x}$-axis point at the companion, and let the coordinate system rotate about the $\hat{z}$-axis with the orbital angular velocity. In these coordinates only the Q_{xx} term contributes to the sum in equation (1). The gravitational potential is

$$\phi(r) = -\frac{GM}{r} - \frac{3}{2}\frac{GQ}{r^3} , \tag{4}$$

where the subscripts on Q have been dropped; $Q = Q_{xx}$.

The transformation to relative coordinates multiplies the potential in equation (4) by M_T/M, where M_T is the total mass of the binary. The relative velocity of a circular orbit is given by $v^2 = r\,d\phi/dr$, where ϕ is the potential in the relative coordinate equation of motion. This gives

$$v^2 = \frac{GM_T}{r}\left[1 + \frac{9}{2}\frac{Q}{Mr^2}\right]. \tag{5}$$

With this, the changes ΔQ, Δv, and Δr are related by

$$2\frac{\Delta v}{v} = -\frac{\Delta r}{r} + \frac{9}{2}\frac{\Delta Q}{Mr^2}, \tag{6}$$

where an unperturbed velocity of $v^2 = GM_T/r$ has been divided out and several small terms have been dropped. The orbital angular momentum is $J = \mu v r$, where μ is the reduced mass, and the orbital period is $P = 2\pi r/v$; these give $\Delta r/r = -\Delta v/v$ and $\Delta P/P = -2\Delta v/v$ at fixed J. These and equation (6) may be combined to give

$$\frac{\Delta P}{P} = -9\left(\frac{R}{a}\right)^2 \frac{\Delta Q}{MR^2}, \tag{7}$$

where R is the radius of the active star and a is the separation. This expression was derived by Applegate & Patterson (1987), and an equivalent expression was derived by Matese & Whitmire (1983).

The physics of the orbital period change is simple. If the active star becomes more oblate, $\Delta Q > 0$, and equation (4) shows that the gravitational field in the equatorial plane of the star gets stronger. If gravity gets stronger, the centrifugal acceleration v^2/r must increase in order to balance gravity. The product rv must remain fixed because the orbital angular momentum does not change, so v increases and r decreases to keep the gravitational and centrifugal accelerations in balance. The orbital period decreases because the stars are moving faster and have a shorter distance to travel to complete an orbit.

2.3. *Deformation by Distortion*

Distortions are deformations that require the magnetic field to contribute to hydrostatic equilibrium in the deformed configuration. Make a distortion by moving material from its initial position $\boldsymbol{x}_0$ to a new position $\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{\psi}$. In general, the displacement $\boldsymbol{\psi}$ can be written as a sum over a complete set of normal modes. In practice, the orbital period of the binary is most sensitive to the quadrupole deformation of the star, so the $l = 2$ modes are the most important ones. The equation of motion for a single such mode is

$$\rho \frac{d^2\boldsymbol{\psi}}{dt^2} = \boldsymbol{L}(\boldsymbol{\psi}) = -\rho\omega^2\boldsymbol{\psi}, \tag{8}$$

where $\boldsymbol{L}(\boldsymbol{\psi})$ is a very complicated differential operator (Unno et al. 1979), and the frequency of an adiabatic quadrupole mode is roughly $\omega^2 \simeq GM/R^3$. If a perturbing force density $\delta\boldsymbol{F}$ distorts the star by statically displacing the mode described by equation (8), the distortion $\boldsymbol{\psi}$ and perturbing force $\delta\boldsymbol{F}$ are related by

$$\delta\boldsymbol{F} = \rho\omega^2\boldsymbol{\psi}. \tag{9}$$

Physically, $-\rho\omega^2\boldsymbol{\psi}$ is the return force of the normal mode, and equation (9) expresses the restoration of hydrostatic equilibrium in the distorted configuration.

A magnetic field affects the shape of a star because the field exerts a force on the currents which maintain it. The current $\boldsymbol{J}$ is related to the field it produces by Ampere's law:

$$\boldsymbol{J} = \frac{c}{4\pi}\nabla \times \boldsymbol{B}. \tag{10}$$

The force on this current is given by Lorentz force law,

$$\boldsymbol{F} = \frac{1}{c}\boldsymbol{J} \times \boldsymbol{B}. \tag{11}$$

Combining these two expressions gives

$$\boldsymbol{F} = \frac{1}{4\pi}(\boldsymbol{B}\cdot\nabla)\boldsymbol{B} - \nabla\left(\frac{B^2}{8\pi}\right). \tag{12}$$

The second term is the familiar gradient of the magnetic pressure. The first term represents the tension in the field lines. This term makes the field lines want to contract along their length. This is the term that makes the field lines act like rubber bands. It is mathematically convenient to write the force law, equation (12), as the divergence of a second-rank tensor. The requisite tensor is the Maxwell stress tensor, defined by

$$T_{ik} = \frac{1}{4\pi}\left[B_i B_k - \frac{1}{2}\delta_{ik}B^2\right]. \tag{13}$$

The magnetic force on the fluid is given by

$$F_k = \nabla_i T_{ik} = \frac{1}{4\pi}(B_i\nabla_i)B_k - \nabla_k\left(\frac{B^2}{8\pi}\right), \tag{14}$$

which is equation (12) written in component form.

The magnetic field in the active star can be written as the sum of a time-average field $\boldsymbol{B}_0$ and a variable field $\delta\boldsymbol{B}$. I include the force due to the average field in the unperturbed configuration and compute the variable distortion due to $\delta\boldsymbol{B}$ with equation (9). The variable field produces a variable Maxwell stress tensor δT_{ik}. Writing $\delta F_k = \nabla_j \delta T_{jk}$ in equation (9) gives

$$\nabla_j \delta T_{jk} = \rho\omega^2\psi_k. \tag{15}$$

The perturbation ΔI_{ik} to the inertia tensor is given by

$$\Delta I_{ik} = \int d^3x\rho(\boldsymbol{x})(x_i\psi_k + x_k\psi_i), \tag{16}$$

where the integral is over the volume of the unperturbed star. This can be computed by mutiplying equation (15) by x_i and integrating over the unperturbed volume to obtain

$$\int d^3x x_i \nabla_j \delta T_{jk} = \omega^2 \int d^3x \rho x_i \psi_k. \tag{17}$$

The divergence can be removed with an integration by parts, giving

$$\omega^2 \int d^3x\rho x_i\psi_k = \int d\hat{S}_j x_i \delta T_{jk} - \int d^3x\, \delta T_{ik}. \tag{18}$$

If the space outside of the star is devoid of plasma, the boundary condition is that $\nabla_j \delta T_{jk} = 0$ both on the boundary of the perturbed star and in the space exterior to the star. The surface integral in equation (18) is over the boundary of the unperturbed star. The terms arising from the difference between applying the boundary condition on the perturbed and unperturbed surfaces are one power of ψ/R smaller than the integrands themselves. Since the integrands in equation (18) are already first order in ψ, and terms of order ψ^2 are being neglected, the boundary condition can be applied on the unperturbed surface. If $\nabla_j \delta T_{jk} = 0$ outside the star, the surface integral in equation (18) can be evaluated with an identity. Integrate $x_i \nabla_j \delta T_{jk} = 0$ over the volume outside of the star. An integration by parts gives

$$\int d\hat{S}_j x_i \delta T_{jk} = \int d^3x \, \delta T_{ik} \, , \tag{19}$$

where the surface integral is over the boundary of the unperturbed star with an inward pointing normal. The surface integral in equation (18) is over the same surface, but with an outward pointing normal. Changing the direction of the normal to the surface changes the sign of the integral. With this, equations (18) and (19) can be combined and the expression symmetrized with respect to the indices i and k to give

$$\omega^2 \Delta I_{ik} = -2 \int d^3x \, \delta T_{ik} \, , \tag{20}$$

where the integral extends over all space.

The energy needed to stabilize the deformation can be calculated from equation (20). The energy density in a magnetic field is $B^2/8\pi = -$ Tr T, where T is the Maxwell stress tensor. Thus, the variable part of the magnetic energy is given by the trace of equation (20),

$$\Delta E_{\rm mag} = -\int d^3x \, {\rm Tr} \, \delta T = \tfrac{1}{2}\omega^2 \Delta \, {\rm Tr} \, I \, , \tag{21}$$

where I is the inertia tensor. If the trace of the inertia tensor varies in the distortion, the variable magnetic energy is of order ψ, and the energy cost of making the deformation is prohibitive. If the trace of the inertia tensor does not change in the deformation, as is the case for a pure quadrupole deformation, the energy cost of stabilizing the displacement is quadratic in ψ, and the Marsh & Pringle (1990) argument, rigorously speaking, does not apply.

The cancellation that occurs in equation (21) if Δ Tr $I = 0$ is possible because the modes being displaced have the character of adiabatic sound waves. The lowest order contribution to the energy in a sound wave vanishes because the contributions from wave crests and troughs cancel. This cancellation can only occur if ψ has spatial nodes. In the case of displacing a quadrupole mode, an oblate deformation can be made by increasing the magnetic energy density at the poles and decreasing it at the equator. These changes cancel if the trace of the inertia tensor does not change. Homologous expansion and contraction is an example of a distortion where this cancellation does not occur; Δ Tr $I \neq 0$ for this mode because the displacement vector does not have any spatial nodes.

The force required to stabilize the displacement is proportional to ψ because the mode is a harmonic oscillator. The mean subsurface field needed to produce a given $\Delta P/P$ can be computed from equations (2), (7), and (20) with Δ Tr $I = 0$:

$$\int d^3x \, \delta T_{xx} = \frac{1}{18} \omega^2 M R^2 \left(\frac{a}{R}\right)^2 \frac{\Delta P}{P} \, . \tag{22}$$

If $\delta T_{xx} \simeq B_0 \delta B/2\pi$ with $\delta B \sim B_0$, the convection zone has half of the volume of the star, and the quadrupole mode in question has $\omega^2 \simeq GM/R^3$, the mean subsurface field is

$$B^2 \simeq \frac{1}{6} \frac{GM^2}{R^4} \left(\frac{a}{R}\right)^2 \frac{\Delta P}{P} \, , \tag{23}$$

which gives $B \sim 2 \times 10^5$ G for V471 Tauri (see § 3.5).

Stabilizing the static displacement of the adiabatic quadrupole mode needed to make the $\Delta P/P \simeq 10^{-6}$ orbital period change in V471 Tau requires the total magnetic energy of the active star to be roughly $E_{\rm mag} \simeq (GM^2/R)(\Delta P/P) \simeq 3 \times 10^{42}$ ergs. If the displaced mode is pure $l = 2$, Δ Tr $I = 0$, and the variable part of the magnetic energy can be as small as $\Delta E_{\rm mag} \simeq (GM^2/R)(\Delta P/P)^2 \simeq 3 \times 10^{36}$ ergs. The active star has brightened by 0.1–0.2 mag since the eclipses were discovered in 1970 (Skillman & Patterson 1988). The luminosity of the active star is 2×10^{33} ergs s^{-1}; the modulation time scale is about a decade, so the energy available for making the distortion is about 10^{41} ergs.

While the strong cancellation allowed by equation (21) is possible for a quadrupole mode, I do not believe that an astrophysically sensible dynamo that takes advantage of it can be constructed. The magnetic field needed to distort the star is very large, and fields of this size are highly buoyant (Parker 1977, 1979) and should rise to the surface of the star rapidly and escape. This buoyant loss of flux requires that the field energy be replaced on the escape time scale, and the star cannot do this. While I cannot prove that a dynamo that utilizes the cancellation in equation (21) does not exist, I do not believe that a 2×10^5 G internal field can be moved about in a star with only a few percent of the magnetic energy being lost each activity period. I conclude that static displacements of adiabatic modes are ruled out as explanations of orbital period modulations.

2.4. *Deformation by Transition*

Transitions are deformations in which the magnetic field causes the star to change from one fluid hydrostatic configuration to another. The quadrupole moment of a rotating star depends on the distribution of angular momentum within the star. In particular, the quadrupole moment is largely determined by the angular momentum carried by the outer layers of the star. These layers dominate because of the r^2 weighting of the mass in the inertia tensor, and because the centrifugal acceleration is largest there. If angular momentum is transferred to the outer part of the star, these layers will spin up and become more oblate, while the inner part of the star will spin down and become less oblate. The net effect is to increase the quadrupole moment of the star because the inner layers are much less important than the outer layers. Thus, the derivative dQ/dJ, where J is the angular momentum of the outer part of the star, is positive.

To estimate dQ/dJ consider a thin shell of mass M_s and radius R rotating with angular velocity Ω in the gravitational potential of a point mass M located at the center of the shell. The shell approximates the small amount of mass in the outer part of the star that determines the quadrupole moment, and the point mass represents the rest of the star. This simple picture is not valid unless $M_s \ll M$. The shell is oblate because of its rotation. The polar radius of the shell is $R_p = R - \psi$ and the equitorial radius is $R_e = R + \psi/2$. If the gravitational potential of the shell is neglected, the requirement that the shell be an equipotential in the combined centrifugal and point mass potential gives the deformation ψ,

$$\frac{\psi}{R} = \frac{1}{3} \frac{\Omega^2 R^3}{GM} \, . \tag{24}$$

The quadrupole moment of the shell is

$$Q = \frac{1}{9} M_s R^2 \left(\frac{\Omega^2 R^3}{GM}\right) . \tag{25}$$

The moment of inertia of the shell is $I_s = \frac{2}{3}M_s R^2$. With this, the derivative dQ/dJ can be evaluated:

$$\frac{dQ}{dJ} = \frac{1}{3}\frac{\Omega R^3}{GM} . \quad (26)$$

The angular momentum transfer ΔJ needed to produce an orbital period change ΔP can be computed by combining equations (7) and (26);

$$\Delta J = -\frac{GM^2}{R}\left(\frac{a}{R}\right)^2 \frac{\Delta P}{6\pi} . \quad (27)$$

The orbital period decreases if the star becomes more oblate, hence the minus sign in equation (27). For the remainder of this paper ΔP and ΔJ will be taken to be the magnitudes of the period change and angular momentum transfer, and the minus sign in equation (27) will be dropped. The energy required to transfer the angular momentum is

$$\Delta E = \Omega_{dr} \Delta J + \frac{(\Delta J)^2}{2I_{eff}} , \quad (28)$$

where $\Omega_{dr} = \Omega_s - \Omega_*$ is the angular velocity of differential rotation, $I_{eff} = I_s I_*/(I_s + I_*)$ is the effective moment of inertia, and quantities with asterisk subscripts refer to the inner part of the star. In the numerical examples below the shell mass is typically $M_s \simeq 0.1M$, so $I_s \approx I_*$, which gives $2I_{eff} \approx I_s$. The transfer of ΔJ to the outer shell will spin the shell up by an angular velocity $\Delta\Omega = \Delta J/I_s$, given by

$$\frac{M_s}{M}\frac{\Delta\Omega}{\Omega} = \frac{GM}{3R^3}\left(\frac{a}{R}\right)^2\left(\frac{P}{2\pi}\right)^2 \frac{\Delta P}{P} . \quad (29)$$

I expect that $\Delta\Omega \lesssim \Omega_{dr}$, although this need not be the case. If the energy requirement, equation (28), is supplied by the nuclear luminosity of the star with no energy storage in the convection zone, the star will be variable with the RMS luminosity variation given by

$$\Delta L_{RMS} = \pi \frac{\Delta E}{P_{mod}} , \quad (30)$$

where P_{mod} is the period of orbital period modulation.

The RMS torque required to periodically exchange ΔJ between the outer shell and the inner part of the star is

$$N = \pi \frac{\Delta J}{P_{mod}} = \frac{\pi}{3}\frac{GM^2}{R}\left(\frac{a}{R}\right)^2 \frac{\Delta P}{P_{mod}} . \quad (31)$$

If this torque is provided by a subsurface magnetic field, the field strength is roughly

$$N \sim \frac{B^2}{4\pi}(4\pi R^2)\Delta R \sim 0.1B^2R^3 , \quad (32)$$

where a lever arm of $\Delta R = 0.1R$ has been assumed. This gives

$$B^2 \sim 10\frac{GM^2}{R^4}\left(\frac{a}{R}\right)^2 \frac{\Delta P}{P_{mod}} . \quad (33)$$

This formula is equation (23) multiplied by a factor $60P/P_{mod}$. Since $P \simeq 1$ day and $P_{mod} \simeq 50$ yr are typical numbers, this factor is typically 3×10^{-3}, which converts the 3×10^5 G fields predicted by equation (23) into fields of several kilogauss. The physics behind this small factor is that the torque in equation (31) can act slowly. The magnetic field does not have any strong unbalanced forces to counteract; it can gradually transfer angular momentum while hydrostatic equilibrium takes care of itself.

The angular momentum transfer, equation (27), is given by observed quantities only. The energy requirement is determined by the angular momentum transfer, the background differential rotation Ω_{dr}, and the mass of the shell. Lowering the differential rotation or raising the shell mass will reduce the energy requirement. The thin shell approximation will break down for M_s much greater than $0.1M$, while equation (29) requires a large variable differential rotation if the shell mass is much smaller than this. In the numerical examples below, the observed orbital period changes can, with one exception, be explained with $\Delta L_{RMS} \approx 0.1L$, $M_s \lesssim 0.1M$, $\Delta\Omega \approx \Omega_{dr} \simeq 0.01\Omega$; the torque can be exerted by a subsurface magnetic field of several kilogauss.

3. NUMBERS

3.1. *O − C Diagrams*

Orbital period modulations are observable as cyclic deviations of observed eclipse times from those calculated from an assumed ephemeris. Let the orbital angular velocity $\omega(t)$ contain a constant term ω_0, a constant period derivative $\dot\omega$, and an oscillatory term:

$$\omega(t) = \omega_0 + \dot\omega t + A \sin \nu t . \quad (34)$$

The orbital phase $\phi(t)$ is given by

$$\phi(t) = \int_0^t dt' \omega(t') . \quad (35)$$

The center of eclipse occurs at the times t_E when the orbital phase is an integral multiple of 2π; $\phi(t_E) = 2\pi E$, where E is the cycle number. The time of eclipses is given by

$$2\pi E = \omega_0 t_E + \frac{1}{2}\dot\omega t_E^2 - \frac{A}{\nu}\cos(\nu t_E) . \quad (36)$$

The correction terms to the constant period ephemeris $t_E = 2\pi E/\omega_0$ are generally small; using this to evaluate the correction terms gives

$$O = t_E = P_0 E + \frac{1}{2} P_0 \dot P E^2 + \frac{AP_0}{2\pi\nu}\cos(P_0 \nu E) , \quad (37)$$

where O stands for the observed eclipse time and $P_0 = 2\pi/\omega_0$. If the eclipse times are computed with the quadratic ephemeris $C = P_0 E + \frac{1}{2}P_0 \dot P E^2$, the oscillatory term in equation (34) produces an oscillation in the $O-C$ diagram of amplitude $O-C = AP_0 P_{mod}/(2\pi)^2$, where $P_{mod} = 2\pi/\nu$ is the modulation period. The oscillatory term in equation (34) produces an orbital period modulation of amplitude $\Delta P/P_0 = A/\omega_0$. The amplitude of orbital period modulation and the amplitude of the oscillation in the $O-C$ diagram are related by

$$\frac{\Delta P}{P} = 2\pi \frac{O-C}{P_{mod}} , \quad (38)$$

where the subscript on the orbital period has been dropped. By analogy to the solar cycle, orbital period modulation should be fairly regular with period P_{mod}, but not strictly periodic.

3.2. *Algol*

The $O-C$ diagram of Algol (Söderhjelm 1980) shows an oscillation with a modulation period of $P_{mod} = 32$ yr and a semiamplitude of $O-C = 0.03$ days, and a feature with $P_{mod} = 180$ yr, and semiamplitude $O-C = 0.15$ days. Equation (38) gives $\Delta P/P = 1.6 \times 10^{-5}$ for the 32 yr feature, and $\Delta P/P = 1.4 \times 10^{-5}$ for the 180 yr feature. Söderhjelm (1980) finds $M_A = 3.6\ M_\odot$, $R_A = 2.9\ R_\odot$, $L_A = 180\ L_\odot$, and $M_B = 0.8\ M_\odot$, $R_B = 3.5\ R_\odot$, $L_B = 7\ L_\odot$. The orbital period is $P = 2.87$ days, the separation is $a/R_B = 4$, and star B is presumably the active star. The orbital period change in the 32 yr cycle is $\Delta P = 4.0$ s. Equation (27) gives $\Delta J = 2.38 \times 10^{48}$ g cm^2 s^{-1} for the requisite angular momentum transfer. If the mass of the shell is $M_s = 0.1 M_B$, the moment of inertia of the shell is $I_s = 6.4 \times 10^{54}$ g cm^2. The energy budget and RMS variability are given by equations (28) and (30); $\Delta E = 8.8 \times 10^{41} + 6.0 \times 10^{43} \Omega_{dr}/\Omega$ ergs, and $\Delta L_{RMS}/L_\odot = 0.71 + 49\Omega_{dr}/\Omega$. The angular momentum transfer and assumed shell mass imply that the variable part of the differential rotation is $\Delta\Omega/\Omega = 1.4 \times 10^{-2}$. More generally, the mass of the shell and the variable part of the differential rotation are constrained by the angular momentum constraint, equation (29), to obey $(M_s/M)(\Delta\Omega/\Omega) = 1.4 \times 10^{-3}$. These numbers imply that the orbital period change in the 32 yr cycle can be explained with $M_s = 0.1 M_B$, $\Omega_{dr} = \Delta\Omega = 0.014\Omega$, and a luminosity variation of $\Delta L_{RMS} = 1.4\ L_\odot = 0.2 L_B$. Equation (33) shows that the torque needed to transfer the necessary angular momentum can be exerted by a subsurface magnetic field of 5.5 kG.

The angular momentum transfer and energy budget of the 180 yr cycle are virtually identical to those of the 32 yr cycle because these two cycles have almost the same $\Delta P/P$. The luminosity variation predicted to accompany the 180 yr cycle is smaller by a factor of $180/32 = 5.6$ and the necessary torque can be applied by a 2.3 kG field. The model can explain the observations.

3.3. *SS Cam*

The $O-C$ diagram of SS Cam (Hall & Kreiner 1980) shows a modulation with a semiamplitude of $O-C = 0.1$ days and a modulation period of $P_{mod} = 2 \times 10^4$ days. This gives $\Delta P/P = 3 \times 10^{-5}$. This binary consists of an F5 star and a K0 star, with the K0 star almost in contact with its Roche lobe (Arnold et al. 1979). The K0 star, presumably the active star, has $M = 2.2\ M_\odot$, $R = 7\ R_\odot$, and $L = 23\ L_\odot$; the binary separation is $a/R = 2.7$, and the orbital period is 4.8 days. The change in the orbital period is $\Delta P = 13$ s. The angular momentum transfer is $\Delta J = 1.32 \times 10^{49}$ g cm^2 s^{-1} according to equation (27). If the mass of the shell is $M_s = 0.1M$, the moment of inertia of the shell is $I_s = 7.0 \times 10^{55}$ g cm^2, and the variable part of the differential rotation is $\Delta\Omega/\Omega = 0.012$. The energy budget and RMS luminosity variation are $\Delta E = 5.0 \times 10^{42}$ ergs and $\Delta L_{RMS} = 9.0 \times 10^{33}$ ergs s^{-1}, where $\Omega_{dr} = \Delta\Omega = 0.012\Omega$ has been assumed. This luminosity variation is 10% of the luminosity of the active star. Equation (33) gives a mean subsurface field of 3.5 kG. The model can explain the observations.

3.4. *SV Cam*

The $O-C$ diagram of SV Cam (Hall & Kreiner 1980) shows an orbital period modulation with a semiamplitude of 0.015 days and a modulation period of $P_{mod} = 3 \times 10^4$ days. Equation (38) gives $\Delta P/P = 3 \times 10^{-6}$; the orbital period is $P = 0.529$ days, so the change in the orbital period is $\Delta P = 0.14$ s. The rest of the system parameters are $M = 0.7\ M_\odot$, $R = 0.9\ R_\odot$, and $L = 0.5\ L_\odot$ for the active star, and the separation is $a/R = 3.6$ (Frieboes-Conde & Herczeg 1973). The angular momentum transfer is $\Delta J = 2.0 \times 10^{47}$ g cm^2 s^{-1}. If the mass of the outer shell is $M_s = 0.1M$, the moment of inertia of the shell is $I_s = 3.7 \times 10^{53}$ g cm^2, which gives the variable part of the differential rotation to be $\Delta\Omega/\Omega = 3.9 \times 10^{-3}$. If $\Omega_{dr} = \Delta\Omega$, the energy budget is $\Delta E = 2.2 \times 10^{41}$ ergs, and the RMS luminosity change is $\Delta L_{RMS} = 0.07\ L_\odot = 0.14L$. The mean subsurface field is 9 kG. The model can explain the observations.

3.5. *V471 Tauri*

This system consists of a 0.8 $M_\odot$ white dwarf and a 0.7 $M_\odot$ main-sequence star in a 12.5 hr orbit. The orbital period (Skillman & Patterson 1988) shows a modulation of semiamplitude $\Delta P/P = 10^{-6}$ and $P_{mod} = 20$ yr if the $O-C$ diagram is interpreted as showing a modulation on top of an overall spin-up of the binary, or $\Delta P/P = 2 \times 10^{-6}$ and $P_{mod} = 40$ yr if all of the orbital period change is due to modulation. The luminosity of the main-sequence star is $L = 0.5\ L_\odot$, the radius of the star is $R = 6 \times 10^{10}$ cm, and the binary separation is $a/R = 4$. The angular momentum transfer is $\Delta J = 1.0 \times 10^{47}$ g cm^2 s^{-1} if $\Delta P/P = 10^{-6}$, and twice this if $\Delta P/P = 2 \times 10^{-6}$. The moment of inertia of the outer shell is $I_s = 2.2 \times 10^{53}$ g cm^2 assuming that the mass of the shell is $M_s = 0.1M$. A relative period change of $\Delta P/P = 10^{-6}$ and a modulation period of $P_{mod} = 20$ yr implies a variable differential rotation of $\Delta\Omega/\Omega = 3.2 \times 10^{-3}$. This implies an energy budget of $\Delta E = 9.3 \times 10^{40}$ ergs and $\Delta L_{RMS} = 0.12\ L_\odot$ if $\Omega_{dr} = \Delta\Omega$. The larger amplitude of modulation and longer period give the same ΔL_{RMS} because ΔE and P_{mod} both double. In their analysis Skillman & Patterson (1988) found that V471 Tau had brightened by roughly 0.15 mag since the eclipses were discovered in 1970. This is consistent with the luminosity variation required to explain the orbital period changes. Skillman & Patterson also found that the photometric wave, presumably due to a large starspot on the active star, followed an ephemeris that implied that the active star rotated 10^{-3} to 2×10^{-3} faster than the orbit. This is consistent with the differential rotation required in the model. The mean subsurface magnetic field is 11 kG. The model fits the data well.

3.6. *RS CVn*

The $O-C$ diagram of RS CVn shows a feature which can be interpreted as a modulation with semiamplitude $O-C = 0.07$ days and modulation period $P_{mod} = 25{,}000$ days. This gives $\Delta P/P = 2 \times 10^{-5}$. The orbital period is 4.8 days, so $\Delta P = 8.3$ s. The parameters of the active star are $R = 3.3\ R_\odot$, $M = 1.36\ M_\odot$, $L = 7\ L_\odot$, and the binary separation is $a/R = 5$ (Popper 1961). The required angular momentum transfer is $\Delta J = 2.4 \times 10^{49}$ g cm^2 s^{-1}. The moment of inertia of the shell is $I_s = 4.4 \times 10^{55}$ g cm^2 with $M_s = 0.1M$. This gives $\Delta\Omega/\Omega = 0.036$. The RMS luminosity variation predicted by the model is $\Delta L_{RMS} = 9.7\ L_\odot$. This is larger than the luminosity of the active star. The model with $M_s = 0.1M$ and $\Omega_{dr} = \Delta\Omega$ cannot explain the orbital period change observed in RS CVn, although the discrepancy is not large and may disappear as the model is improved. For example, the energy requirement and luminosity variation may be overestimated by a factor of a few in equations (28) and (30). The choices $\Omega_{dr} = \Delta\Omega$ and $I_s = I_*$ imply that the two terms in equation (28) contribute equally. The energy requirement is lowered by a factor of 2 if $\Omega_{dr} = 0$, although there is no particular reason that RS CVn should be oscillating about solid body rotation and the other stars oscillating about a nonzero background differential rotation.

Furthermore, I have assumed that all of the energy variation ΔE appears as luminosity variation, and this need not be the case. If the angular momentum exchange is between a convective envelope and a radiative inner part of the star, the rotational kinetic energy dissipated in the radiative part will not contribute to the luminosity variation because radiative zones cannot respond faster than the local Kelvin-Helmholtz time, and this is much longer than the modulation period. In addition, some storage of energy in the convection zone may occur. If the energy dissipated in the inner part of the star is omitted from the luminosity variation, then $I_{\rm eff} = I_s$ in equation (28), and the second term in ΔE drops by a factor of 2. These two corrections lower the luminosity variation by a factor of 4, which gives $\Delta L_{\rm RMS} = 2.4\ L_\odot$, which is an acceptable, though clearly strained, fit to the data. The implied subsurface magnetic field is 13 kG. The model can plausibly explain the orbital period modulation in RS CVn, although the fit is not as good as for the other stars and requires special assumptions. There is no apparent reason why the model parameters $M_s = 0.1M$, $\Omega_{\rm dr} = \Delta\Omega = \Delta J/I_s$, and $\Delta L_{\rm RMS} \simeq 0.1L$, which work well for the other stars, fail for RS CVn.

4. DISCUSSION AND CONCLUSIONS

I have presented a model which explains the orbital period modulations seen in some eclipsing binaries as a manifestation of stellar magnetic activity. Gravitational quadrupole coupling provides the mechanism by which the orbit responds to changes in the internal structure of an active star. I have analyzed two mechanisms by which magnetic activity can change the quadrupole moment of a star, and found that the requisite shape changes can be produced by the cyclic exchange of angular momentum between the inner and outer parts of the star. The torque needed to transfer the angular momentum can be exerted by a subsurface magnetic field of several kilogauss. The model typically requires variability at the $\Delta L/L \simeq 0.1$ level and a variable differential rotation of $\Delta\Omega/\Omega \simeq 0.01$ to explain a $\Delta P/P \simeq 10^{-5}$ orbital period modulation. The model provides a good fit to Algol, SS Cam, SV Cam, and V471 Tauri, and a strained fit to RS CVn. Since the model proposes that magnetic activity is the underlying cause of orbital period modulation, and magnetic activity requires convection, the model predicts that orbital period modulation should only occur in systems with a least one convective star. This is in agreement with the study of 101 Algols by Hall (1989), who found that all 31 systems which showed evidence of period modulation contained a convective star.

This mechanism has several testable predictions. The luminosity variation is required to have the same period as the orbital period modulation. In addition, any other indicator of magnetic activity (starspot amplitude, coronal X-ray luminosity, emission cores in Ca II or Mg II lines, etc.) should also show this period. The luminosity variation should be entirely due to a temperature variation since large changes in the radius of the star are ruled out by energetics. The star should be hottest, and thus bluest, when it is most luminous.

The relative phases of the variations carry important information. Orbital period minimum always occurs at quadrupole moment maximum, and quadrupole moment maximum always occurs when the outside of the star is spinning the fastest. Extrema of the $O-C$ diagram occur 90° in phase after orbital period extrema. The phase of the luminosity variation depends on the sense of the differential rotation. The energy stored in rotation is largest when the star is furthest from solid body rotation. If the sense of the background differential rotation is that the outside of the star spins faster than the inside, then the rotational energy is largest when the outside spins the fastest, which occurs at orbital period minimum. The luminosity variation is $\Delta L = -dE_{\rm rot}/dt$, so the luminosity maximum lags the rotational energy maximum by 90° in phase. If the background differential rotation has the outside of the star spinning faster than the inside, then the luminosity and $O-C$ diagram are 180° out of phase; luminosity maximum occurs at $O-C$ minimum. If the background differential rotation has the inside spinning faster than the outside, the star is closest to solid body rotation at orbital period minimum. In this case the luminosity and $O-C$ diagram are in phase.

Luminosity variation, temperature variation, and orbital period modulation have all been detected in the binary CG Cygni by Hall (1991). Hall finds that the periods of the luminosity variation and the orbital period modulation are the same, and that the luminosity variation can be accounted for by a temperature change. He also finds that the luminosity maximum coincides with the $O-C$ minimum, and concludes that the sense of the differential rotation in the active star is that the outside rotates faster than the inside. Hall's result is strong observational support for the mechanism which I propose.

A plausible, but by no means unique, picture of the angular momentum exchange is as follows: In the absence of a magnetic field, the level of differential rotation in a star is given by the balance between a driving torque and the damping due to friction. The presence of a magnetic field can alter this balance and change the differential rotation of the star. A strong magnetic field will tend to enforce uniform rotation because differential rotation stretches field lines by shearing them, and this costs energy. When the field is strong, the inner and outer parts of the star will be forced toward solid body rotation. When the field is weak, the differential rotation will relax back towards its zero field value. In this simple picture the cyclic waxing and waning of the subsurface magnetic field causes the cyclic redistribution of angular momentum by changing the balance between the driving and damping of differential rotation in the star.

The sense of the differential rotation, the magnitude of the subsurface field, and the relative phase between the subsurface field strength and surface magnetic activity can be determined if the simple picture described above is correct. The sense of the differential rotation can be determined from the relative phase of the luminosity and $O-C$ diagram variations. If the sense of the differential rotation is that the outside of the star rotates faster than the inside, as is the case for CG Cygni, then subsurface field minimum should coincide with orbital period minimum. If surface magnetic activity is in phase with subsurface field strength, the star should be the most magnetically active when its orbital period is the longest and least active when its period is the shortest. If the surface activity is strongest when the subsurface field is changing the fastest, the orbital period and surface activity should be 90° out of phase. A measurement of the relative phase between subsurface field strength and surface magnetic activity will be a very important constraint on dynamo models of magnetic activity.

This project is part of a collaboration with Joe Patterson, and I have learned a great deal from him in the course of it. Kevin Prendergast and Dick McCray were wonderful sounding boards for many of the ideas in this paper. I have also had very useful conversations with Doug Hall, Ed Spiegel, and Stirling Colgate. Most of the material on the relative phases between orbital period and luminosity changes were worked out with Doug Hall. This is contribution number 462 of the Columbia Astrophysics Laboratory.

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